ON THE STRUCTURE OF p-STABLE AGGREGATION OPERATORS

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The aim of the paper is to study the structure of p-stable aggregation operators which were introduced in [2] for controlling output errors in aggregation process.

Definition 1. Let $n \in \mathbb{N}$, $n \geq 2$ and $p \in [1, \infty]$. An n–ary aggregation operator is p–stable if for all $\mathbf{x} = (x_1, \dots, x_n)$ and $\mathbf{y} = (y_1, \dots, y_n) \in [0,1]^n$ we have

$$|\mathbf{A}(\mathbf{x}) - \mathbf{B}(\mathbf{y})| \leq \|\mathbf{x} - \mathbf{y}\|_p, \tag{1}$$

where $\|.\|_p$ is the standard norm in the Banach space $\mathbb{R}^n$.

If we work with a fixed number n of inputs we will use only the name p–stable aggregation operator.

Since for $p \in [1, \infty[$ and $\mathbf{u} = (u_1, \dots, u_n) \in [0,1]^n$ the norms are $\|\mathbf{u}\|_p = \left(\sum\limits_{i=1}^{n} |u_i|^p\right)^{1/p}$, and $\|\mathbf{u}\|_\infty = \max\limits_{1 \leq i \leq n} |u_i|$, 1–stable aggregation operators are just 1–Lipschitz aggregation operators, and ∞–stable aggregation operators coincide with so–called kernel aggregation operators introduced and studied in [4,5].

Denote by $\mathcal{S}_p$, $p \in [1, \infty]$, the class of all p–stable aggregation operators. Then the system $(\mathcal{S}_p)_{p=1}^{\infty}$ is decreasing with respect to the parameter p:

$$\forall\, p, q \in [1, \infty] \quad [\ p < q \quad \Rightarrow \quad \mathcal{S}_p \subset \mathcal{S}_q\ ].$$

Due to the nested structure of the system $(\mathcal{S}_p)_{p=1}^{\infty}$, kernel aggregation operators are p–stable for all other $p \in [1, \infty[$. For composed aggregation operators $\mathbf{D}$ which are defined by the composition of aggregation operators in the sense $\mathbf{D} = \mathbf{A}(\mathbf{B}, \mathbf{C})$ we can prove that if $\mathbf{B}$, $\mathbf{C}$ are $p-$ and q–stable aggregation operators respectively, and $\mathbf{A}$ is a kernel aggregation operator then $\mathbf{D}$ is certainly r-stable, where $r = \min(p, q)$. This property of composed operators is not guaranteed by any $\mathbf{A} \in \mathcal{S}_p$ with $p < \infty$. It can be shown that for each $p < \infty$ there exist kernel aggregation operators $\mathbf{B}$, $\mathbf{C}$ and a p–stable (binary) aggregation operator $\mathbf{A}$ such that $\mathbf{D} = \mathbf{A}(\mathbf{B}, \mathbf{C}) \notin \mathcal{S}_1$.

There are also other reasons why the class of all ∞–stable aggregation operators is a distinguished class of aggregation operators. We prove some results on p–stable aggregation operators, in general, and then we focus our study on the structure of these aggregation operators in their binary form.

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