OPTIMAL INTERPOLATORY AND OPTIMAL SMOOTHING SPLINES USING B-SPLINE REPRESENTATION

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Spline functions have proved to be very useful in numerical analysis and statistics and have found applications in science, engineering and economics.

Splines are very often used in the interpolation problem or in the smoothing problem. It is well known that the natural splines of odd degree $2m-1$ interpolating given function values in their knots have a minimal L_2-norm of its m-th derivative on the class of interpolants from W_2^m. Similarly the natural splines of even degree $2m$ interpolating given mean values have a minimal L_2-norm of its m-th derivative on the class of interpolants from W_2^m.

These extremal properties were used in the construction of smoothing splines, which are very useful in the situations when there are given inexact data. In this case minimized functional consists from L_2-norm of the corresponding derivative and the least square component. There is strong connection between the degree of spline and the order of the derivative in the first component of the minimized functional. Other strong connection is the knowledge of the function values in all knots of spline and natural boundary conditions which must be satisfied.

In this paper we will consider the class of splines of degree $k > 0$ only. We will separate the knots of spline and the interpolatory points and we will not require an accomplishment the natural boundary conditions so we will have free parameters which we will can use to optimization purposes. First we will solve this problem: we will use free parameters of such splines of degree $k > 0$ which interpolates the prescribed data to minimize some functionals with geometrical or physical meaning (curvature, energy). In the second part of this paper we will use these functionals for the construction of the smoothing splines.

To get this interpolatory or smoothing spline we shall use the B-spline basis and a generalized inverses of some matrix. We shall give the conditions under which this interpolatory or smoothing spline exists and it is unigue.