HYPERCYCLIC SUBSPACES

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A vector x in a Hilbert space $\mathcal{H}$ is called hypercyclic for a bounded linear operator $T : \mathcal{H} \to \mathcal{H}$ if the orbit

$$\{T^n x : n \geq 0\}$$

is dense in $\mathcal{B}$. F. León-Saavedra and A. Montes-Rodríguez [1] proved that if T satisfies the Hypercyclicity Criterion and the essential spectrum of T intersects the closed unit disk, then there is an infinite-dimensional closed subspace of $\mathcal{H}$ consisting, except for zero, entirely of hypercyclic vectors for T. The converse is true even if T is a hypercyclic operator which does not satisfy the Hypercyclicity Criterion. As a consequence, other characterizations are obtained for an operator T to have an infinite-dimensional closed subspace of hypercyclic vectors. The essential minimum modulus will pay a important role in an argument that depends in an essential way on a deep theorem of Zemánek that relates the modulus with the distance from zero to the left essential spectrum.

These results apply to most of the hypercyclic operators that have appeared in the literature. In particular, they apply to bilateral and backward weighted shifts, perturbations of the identity by backward weighted shifts, multiplication operators and composition operators. The main result also applies to the differentiation operator and the translation operator $T : f(z) \to f(z+1)$ on certain Hilbert spaces consisting of entire functions.

REFERENCES

[1] F. León-Saavedra and A. Montes-Rodríguez, *Spectral theory and hypercyclic subspaces*, Trans. Amer. Math. Soc. **353** (2001), 247-267.