QUADRATIC HAMILTONIANS, QUANTUM CORRELATION FUNCTIONS AND ORBITS OF WEYL-HEISENBERG GROUP

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Monomials and homogenous polynomials in quantum positions and momenta are considered. From the fact that the commutator of the quadratic Hamiltonian and a monomial of order m is a homogeneous polynomial of order m it is deduced that the time evolution of a monomial in Heisenberg picture for a quadratic Hamiltonian is a homogenous polynomial of the same order. It follows that the time evolution of mean values of a monomial and also of arbitrary quantum correlation function can be explicitly computed if we know the mean values of certain monomials in the initial state (if, again, the Hamiltonian is quadratic).

It is then shown that the evolution operators (for quadratic Hamiltonians) and the usual Weyl operators in a sense "commute". It follows that the image of arbitrary orbit of Weyl-Heisenberg group under the time evolution of a quadratic Hamiltonian (for a fixed time t) is again an orbit of Weyl-Heisenberg group .

Finally, the quadratic Hamiltonian is specified to be the Hamiltonian of quantum free particle. It is shown that if the initial state of the free particle is on an orbit of Weyl-Heisenberg group then the time evolution never intersects this orbit again. A simple consequence is that for sufficiently large number there always exist two different orbits of Weyl-Heisenberg group such that the (constant) value of the dispersion of some quantum position is the given number for both orbits.