DERIVATIVE OF FUZZY REAL FUNCTIONS

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Fuzzy real function is a mapping from R to a set of all fuzzy real numbers. A simple approach on diferentiation of these fuzzy real functions based on α-level functions is studied. Some properties of this derivative are shown, including relationship of the derivative and the monotonicity of fuzzy function and also equivalents of standard theorems such as Rolle theorem are proven.

Definition. $(f'(x))_\alpha$ denotes the derivative of fuzzy function f on the level α at the point x.

$$(f'(x))_\alpha = [I_\alpha; S_\alpha],$$

where

$$I_\alpha = \inf\{f'_\beta(x); |\beta| \geq \alpha\},$$

$$S_\alpha = \sup\{f'_\beta(x); |\beta| \geq \alpha\},$$

for $\alpha \in]0;1]$ and $|\beta| \in]0;1]$.

$$I_0 = \inf_{0<\alpha\leq 1} I_\alpha, \text{ a } S_0 = \sup_{0<\alpha\leq 1} S_\alpha.$$

Definition. Derivative of fuzzy function f at point $x \in]a,b[$ is fuzzy set $f'(x)$ defined as follows:

$$\mu_{f'(x)}(z) = \sup\{\alpha; z \in [I_\alpha, S_\alpha]\}$$

where $[I_\alpha; S_\alpha]$ are derivatives on level α of fuzzy function f at point x, assuming $\sup\emptyset = 0$.

References

[1] Kalina, M. *Derivatives of Fuzzy Functions and Fuzzy Derivatives*, Tatra Mountains Mathematical Publications 12, 1997.

[2] Dubois, D., Prade, H. *Towards Fuzzy Diferential Calculus. Part 1-3*, Fuzzy Sets and Systems 8, 1982.

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