HAHN DECOMPOSITION OF MODULAR MEASURES AND APPLICATIONS

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Let L be a lattice effect algebra and X a Banach space. A function $\mu: T \to X$ is said to be a modular if $\mu(a \vee b) + \mu(a \wedge b) = \mu(a) + \mu(b)$ for all $a, b \in L$ and it is said to be a measure if $\mu(a \oplus b) = \mu(a) + \mu(b)$ for all orthogonal $a, b \in L$. A measure is (σ)-order continuous if $\mu(a_n) \to \mu(a)$ whenever (a_n) is a monotone net (sequence) converging to a.

We prove a Hahn decomposition theorem for real-valued σ-order continuous modular measures on σ-complete lattice effect algebras. As an application, we establish a natural isomorphism between the space of all bounded real-valued modular measures μ on L and the space of all real-valued order continuous modular measures on $C(\tilde{L})$; here $\tilde{L}$ is a suitable completion of a quotient of L and $C(\tilde{L})$ the center of $\tilde{L}$. This allows us to prove the following result. If X is a Banach space with the Radon-Nikodym property and μ is a modular measure of bounded variation then $\overline{\mu(L)}$ is compact. If, in addition, μ is nonatomic then $\overline{\mu(L)}$ is also convex. In the Boolean case, this theorem is known as Uhl's theorem [3].

Moreover, if X is a B-convex Banach space and μ is a nonatomic modular measure of bounded variation then we show that $\overline{\mu(L)}$ is likewise convex. This is another generalization, this time of V. M. Kadets' result valid in the Boolean case [2].

References

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