STATES ON PSEUDOEFFECT ALGEBRAS

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Pseudoeffect algebras – or PE-algebras for short – have been introduced in [1] as a common generalization of effect algebras on the one hand and pseudo-MV algebras on the other hand. That is, PE-algebras are partial additive structures $(E; +, 0, 1)$ which fulfil similar axioms as effect algebras; but they are, like pseudo-MV algebras, possibly noncommutative. Because of the close connection of effect algebras to the formalism of quantum mechanics, PE-algebras are considered as quantum structures; and since they include pseudo-MV algebras and MV-algebras as subclasses, they are also a member of the family of algebras providing semantics for many-valued logics.

To understand the structure of a PE-algebra is not an easy matter. In [2], congruence relations were studied; in particular, homomorphisms to the real unit interval, that is, *states* were considered. We give here a further contribution to the problem of how to enlighten the structure of a PE-algebra on the basis of its state space.

States are homomorphisms to a commutative algebra and thus describe the commutative part of a PE-algebra only. Not amazingly, in general there need not be a state at all; this in fact already applies to effect algebras [3]. So the probably first and most basic question here is under which conditions states exists.

To this end, we concentrate on those PE-algebras for which the consideration of states primarily make sense: on intervals of partially ordered groups which are the lexicographic product of an abelian group and a possibly nonabelian group. In this situation, states are the right tool to describe exactly the structure of the first component.

We introduce a certain algebraic condition to define *nearly commutative* PE-algebras. We will see that, under the assumption of a certain Riesz property, the PE-algebras in which we are particularly interested fall into this category. We will finally prove that every nearly commutative PE-algebra possesses at least one state.

This work has been supported by the VEGA grant 2/7192/20 (Slovak Republic).

References

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