STRUCTURE THEORY FOR PSEUDO-EFFECT ALGEBRAS

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Recently [2], pseudo-effect algebras were introduced as a non-commutative generalization of effect algebras. The latter type of algebra is modeled upon an interval in the partially ordered group of self-adjoint operators of a Hilbert space; the positive operators of a Hilbert space lying below identity form in a natural way the standard effect algebra. Pseudo-effect algebras arise in a similar manner from partially ordered groups, but this time not necessarily from abelian ones; any unit interval in a positive cone gives rise to a pseudo-effect algebra.

We are working towards a structure theory for the new type of algebra. First of all, the question arises under which exact algebraic conditions a pseudo-effect algebra is representable by means of a . This problem found certain partial solutions; so in [3], a condition comparable to the Riesz Decomposition Property as known for s, was proved to be sufficient.

We are here concerned with the analysis of a pseudo-effect algebra by studying its quotients, and in particular its states. We develop a theory of congruences and ideals, proceding in a similar manner as it was done for the commutative case [4],[1].

Concerning the "roughest" quotients of pseudo-effect algebras, the states, the following turns out to be true. Any group representable pseudo-effect algebra possesses a commutative quotient such that the set of states of the latter remains in a natural sense unchanged. This result was to be expected insofar, as the real unit interval is a commutative structure, and it parallels a similar theorem about locally compact groups and their characters. It means roughly that the study of states is reducible to the commutative case.

References

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