OUTLIERS IN DATA ENVELOPMENT ANALYSIS

František Sudzina

Faculty of Science
P.J.Šafárik University
Jesenná 5, 04154 Košice, Slovak Republic
sudzina@iname.com

It is fairly easy to evaluate profit-making companies. But it is sometimes necessary to compare decision making units (DMUs) although their inputs and outputs cannot be evaluated in means of market prices. Data Envelopment Analysis (DEA) is a linear programming based technique for measuring the relative performance of organizational units where the presence of multiple inputs and outputs makes comparisons difficult. The aim of this paper is not to present the way to evaluate the best practice, the worst practice, but the way to identify outliers in Data Envelopment Analysis. Outliers will be discussed because they may be considered to be relatively efficient only because they lie in extreme positions and they eventually bias the analysis.

When we run DEA, DMUs, which have relative efficiency equal to 1, represent the best practice. But there may be a task to find out which DMUs are least efficient. DEA efficiency score cannot be used because they are best possible since the weights are picked with the goal to show the unit in the best light. The reason to run this kind of research could be the intention to change a management of a state-owned or a non-profit organization or to find out which countries are doing the worst job in complying with Maastricht criteria.

In order to identify the worst DMUs, we can run reverse DEA. The least efficient units will have scores equal to 1. The score of a target unit can be obtained as a solution of a reverse DEA model, calculated as a DEA model where inputs are exchanged with outputs and vice versa. This way a negotiator knows beforehand whether it is possible that his DMU can be considered as the worst one when the weights are applied, which are the worst from his point of view. If so, he must invest all the effort into negotiating the best weights. If not, he does not have to be that much cautious about the negotiations. So, it is necessary to take some time and to find out what is wrong with DMUs, which have relative efficiency equal to 1 because they represent the worst practice. Outliers are discussed in this paper because they can eventually bias the analysis. Especially in a case when DEA is used to filter out inefficient DMUs and efficient DMUs are used for another analysis. Outliers are relatively efficient DMUs which use the fewest amount in at least one input component (input-oriented model) or produce the highest amount in at least one output component (output-oriented model). So, they are efficient mainly because they lie in extreme positions. It is often the case that an analysis is ran without

checking, which of the efficient DMUs are outliers. All DMUs, which are in the best and the worst practice groups, are outliers. But there may be also other outliers, which cannot be identified this way. A DMU should be defined as an outlier, if it shows the worst practice from all the relativelly efficient DMUs.
Another reason for identification of outliers is that there can be done two different studies of a certain system and then it can turn up that the results are a little bit different. For example, there was done a Data Envelopment Analysis of Slovak banks[4]. It showed that among others, Všeobecná úverová banka and Komerčná banka were efficient. This seemed really strange. Specially because we knew the situation plus [1] suggested the Všeobecná úverová banka was not efficient and Komerčná banka was not efficient when Austrian banks were added to the analysis in [4]. The reason for this disproportion was the fact that the two banks were outliers. This is why every study which uses DEA should always explicitelly say which DMUs are outliers.

References

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