PSEUDO HENSTOCK-KURZWEIL INTEGRAL

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Pseudo-integral based on generalized pseudo-operations, operations that need not be commutative nor associative, has been introduced by Pap and Štajner-Papuga. Its construction is similar to construction of Riemann integral through the use of Riemann sums, but the classical operations and measure are substituted by certain class of generalized pseudo-operations and related pseudo-measure. It is proven that this pseudo-integral has the following form:

$$\int_{[a,b]}^{(\oplus,\odot)} f\, d\mu_p = k^{-1}\Big(\int\limits_a^b (k(f(t)))^{\gamma}\, dt\Big).$$

Operations needed for this construction have been introduced and investigated by Pap and Vivona.

The Henstock-Kurzweil integral has been introduced in 1950's as a generalized version of Riemann integral by J. Kurzweil.

Definition 1. *It is said that f is Henstock-Kurzweil integrable on $[a,b]$ whenever there is a real number I satisfying the following condition: for each $\varepsilon > 0$ there exists a positive function δ defined on $[a,b]$, such that*

$$|\sum_{i=1}^{p} f(x_i)\mu(A_i) - I| < \varepsilon,$$

for each δ-fine partition P of $[a,b]$ and μ is Lebesque measure.

The number I from previous Definition is called the Henstock-Kurzweil integral of f on $[a,b]$.

Since in the cores of pseudo-integral and Henstock-Kurzweil integral are some forms of the Riemann sum, generalized pseudo-operations and pseudo-measure from the construction of pseudo-integral have been applied to the construction of Henstock-Kurzweil integral. Now, new integral known as pseudo Henstock-Kurzweil integral, as well as some of its properties, can be presented.