ADAPTIVE FINITE VOLUME METHOD SCHEMES IN PROCESSING OF 3D DATA

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We propose the coarsening strategy for finite volume computational method given by Mikula and Ramarosi ([4]) for the numerical solution of the modified (in the sense of Catté, Lions, Morel and Coll) Perona–Malik nonlinear image selective smoothing equation (called *anisotropic diffusion* in the image processing) aaplied to 3D data.

Our 3D data can be modelled as a real function $u_0(x)$ representing values of greylevel intensity, defined in some 3D rectangular subdomain $\Omega \subset \mathbb{R}^d$. Basic tasks of image processing and computer vision like selective image smoothing, enhancement, restoration, segmentation, edge detection, shape analysis, optic flow computations etc. are recently modelled by application of evolutionary PDE to $u_0(x)$. Such approach is known as *image multiscale analysis* ([1],[2],[3]) since the initial image $u_0(x) = u(0,x)$ is associated with a sequence of images $u(t,x)$, depending on an abstract parameter $t > 0$ called scale. The well known examples are nonlinear diffusion equations of Perona-Malik type ([5]), and and generalized mean curvature flow equations.

In the finite volume method, the discrete approximations are piecewise constant in control volumes. As the solution tends to be more flat in some regions of the 3D data with the increasing scale, we can improve the efficiency of the method using adaptivity, i.e. choosing irregular grid with less finite volumes. Process of merging finite volumes is called coarsening in numerical methods for solving PDEs.

In this paper we present a method for creating an irregular adaptive grid, and adjustment of a numerical scheme to such a grid. We explored semi-implicit and exlicit schemes and introduce some computational comparisons and results of numerical experiments applied to real and artificial images.

References

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