THREE ESTIMATORS OF COVARIANCE FUNCTION

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We will consider a linear regression model

$$\mathbf{X} = \mathbf{F}\beta + \epsilon \quad E(\epsilon) = 0 \quad Var(\epsilon) = \Sigma$$

Under condition of stationarity of covariance function we can write:

$$\Sigma = \sum_{i=1}^{p} \nu_i V_i = \sum_{i=1}^{n} R(i-1)U_i$$

where matrix $(U_i)_{k,l} = 1$ for k-l=i-1 and $(U_i)_{k,l} = 0$ otherwise. The aim of this article is to compare three forms estimations of parameters of covariance function.
1. Nonparametric estimator value R(t), where t=1, ..., n-1 in the form:

$$\widehat{R}(t) = \frac{1}{n-t} \sum_{i=0}^{n-1} (X(i+t) - \widehat{X}(i+t))((X(i) - \widehat{X}(i))$$

2. MINUE estimator function $g(\nu) = \lambda'\nu$ based on matrix S, where S in the form $S_{i,j} = \widehat{R}(|i-j|)$, matrix $V = V_1 + ... + V_p$ is regular and

$$\widehat{g}(\nu) = Tr(\sum_{i=1}^{p} \alpha_i V^{-1} V_i V^{-1} S)$$

where $\alpha = (\alpha_1, ..., \alpha_p)$ is a solution a linear system $M\alpha = \nu, M_{i,j} = Tr(V_i V^{-1} V_j V^{-1})$ for i, j = 1 ,...,p
3. The approximate least squares estimator $\tilde{\nu}$ based on method due to Box. The function R(t) is assumed to be know, continuous and twice continuosly differetiable in ν

$$\widehat{R}(t) = R(t) + (\widehat{R}(t) - R(t)) = f(t, \nu) + \eta(t)$$