LINEAR REPRESENTATIONS OF TRIANGLE GROUPS AND CONGRUENCE SUBGROUPS

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Triangle groups $T(2,m,n) = \langle x, y | x^n = y^2 = (xy)^m = 1 \rangle$ are well known and important in various areas of mathematics, such as group theory, Riemann surfaces, and topological graph theory. The objective of the latter is the study of maps, that is, graphs embedded in surfaces. A particularly interesting class of maps are regular maps, in which the group of (orientation-preserving) automorphisms acts regularly on darts. It turns out that regular maps of type $\{m, n\}$, that is, of valence n and covalence m, are in one-to-one correspondence with torsion-free normal subgroups (or, equivalently, homomorphic images) of $T(2,m,n)$.

Historically, a prominent example of a regular map is the famous Klein map of type $\{3,7\}$ on an orientable surface of genus 3, whose automorphism group is isomorphic to $PSL_2(7)$. Using congruence subgroup of $T(2,3,7)$ we obtain an interesting three-dimensional representation of the Klein map.