CONNECTEDNESS APPLIED TO CLOSURE SPACES AND STATE PROPERTY SYSTEMS

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In [5] an equivalence of the categories **SP** and **Cl**s was presented. The category SP consists of the state property systems [4] and their morphisms, which are the mathematical structures that describe a physical entity by means of its states and properties within the Geneva-Brussels approach to the Foundations of Physics [1,2,3,6,7,8]. For a state property system $(\sum, L, \xi)$ we have that $\sum$ is a set, representing the states of the physical entity, L a complete lattice, representing the properties of the physical entity, and ξ a map from $\sum$ to $P(L)$, where, for $p \in \sum$ we have that $\xi(p)$ is the set of actual properties the entity being in state p. The category **Cl**s consists of the closure spaces and the continuous maps.

In earlier work we have shown, using the equivalence between **Cl**s and **SP**, that some of the axioms of quantum axiomatics are equivalent with separation axioms on the corresponding closure space. More particularly we have proven that the axiom of state determination is equivalent to the T_0 separation axiom [5] and the axiom of atomicity is equivalent to the T_1 separation axiom [9]. In the present paper we analyse the intimite relation that exists between classical and nonclassical in the state property systems and disconnected and connected in the corresponding closure space.

In order to define the concept of classical into the category **SP**, we introduce super selection rules and classical properties for state property systems. For $a, b \in L$ we say that a and b are separated by a super selection rule ($a\ ssr\ b$) iff for $p \in \sum$ we have:

$$a \vee b \in \xi(p) \Rightarrow a \in \xi(p) \text{ or } b \in \xi(p).$$

A property $a \in L$ is a classical property if there exists a property $a^c \in L$ such that $a \vee a^c = I$, $a \wedge a^c = 0$ and $a\ ssr\ a^c$. A closure space $(\sum, F)$ is connected if the only clopen (i.e. closed and open) subsets of $\sum$ are the empty set $\emptyset$ and $\sum$ itself.

We prove that the classical properties of a state property system $(\sum, L, \xi)$ correspond exactly to the clopen subsets of the corresponding closure space $(\sum, K(L))$:

$$a \text{ is classical} \Leftrightarrow K(a) \text{ is clopen.}$$

Thus connected closure spaces correspond precisely to state property systems for which the elements 0 and I are the only classical properties, the so called pure nonclassical state property systems. We prove that a state property system can be decomposed into its pure nonclassical components over a classical state property system. This reflects the decomposition theorem of generalized quantum structures into irreducible quantum structures over a center for the case of state property systems. For topological spaces, every closure space can be decomposed uniquely into connected components. We investigate in which way the decomposition for state property systems gives rise to a decomposition for general closure spaces.

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