LYAPUNOV'S THEOREM FOR MEASURES ON D-POSETS

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Let P be a D-poset and $\mu = (\mu_1, \dots, \mu_n) : P \to^n$ a measure.

DEFINITION *μ is strongly continuous if for every $\varepsilon > 0$ there are $\{a_1, \dots, a_m\}$ with $1 = \oplus_{i=1}^m a_i$ and $|\mu(b)| < \varepsilon$ for $b \leq a_j$, $j \leq m$.*

We prove

THEOREM *Suppose P with the interpolation property (a property weaker than σ-completeness) and μ positive and strongly continuous, then for all $a, b \in P$ with $a \perp b$ or with $a \leq b$ the segment $\overline{\mu(a), \mu(b)}$ is contained in the range; in particular the range is a star-shaped domain with respect to 0.]*

COROLLARY 1 *Let P a D-lattice with the interpolation property and $\mu : P \to^n$ a strongly continuous measure which is modular. Then $\mu(P)$ is convex.*

COROLLARY 2 *If P is σ-complete and $\mu : P \to^n$ an atomless σ-order continuous measure which is modular. Then $\mu(P)$ is convex.*

It follows from Corollary 1 observing that under these assumptions atomless is equivalent to strongly continuous.