PSEUDO-EFFECT ALGEBRAS

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Recently, pseudo-MV algebras have been introduced as a non-commutative analogue to MV-algebras [1], which in turn arise in a canonical manner from the unit interval $G[0, u]$ of an ℓ-group G with strong unit u.

Effect algebras on the other hand were introduced to model algebraically the unit interval of the additive group of bounded self-adjoint operators of a Hilbert space and are in this sense of major importance in quantum physics.

In a similar manner as it was done for pseudo-MV algebras on the base of MV algebras, we introduce a new type of structure called *pseudo-effect algebras* by dropping commutativity from the axioms for effect algebras. More precisely, a structure $(E; +, 0, 1)$, where $+$ is a partial binary operation and 0 and 1 are constants, is called a pseudo-effect algebra if the following axioms hold in it:

(E1) $a + b$ and $(a + b) + c$ exist, iff $b + c$ and $a + (b + c)$ exist, and in that case $(a + b) + c = a + (b + c)$.

(E2) For each a there is exactly one b and one c such that $a + b = c + a = 1$.

(E3) If $a + b$ exists, then there is a c and a d such that $c + a = b + d = a + b$.

(E4) If $a + 1$ or $1 + a$ exists, then $a = 0$.

Clearly effect algebras, but also pseudo-MV algebras are included here.

We work towards a structure theory for the new kind of algebras. Unfortunately, even in the case of the commutative effect algebras not much is known in that respect for the general case. But under the additional assumption that the Riesz decomposition property holds, it has been proved that an effect algebra is representable as the unit interval of a group in which also the decomposition property holds [2].

It turns out that an analogous theorem holds for pseudo-effect algebras: Every such structure fulfilling a certain Riesz-like property may be represented as the unit interval of a partially ordered - i.e. not necessarily abelian or lattice ordered - group.

The theorems of that kind for effect algebras [2], pseudo MV-algebras [3] and also on Bosbach's semiclans [4] can be reproved.

REFERENCES

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