NON-CONTINUOUS T-NORMS WITH CONTINUOUS DIAGONALS

Dana Smutná

Faculty of Natural Science, Matej Bel University
Department of Mathematics
Tajovského 40, 974 01 Banská Bystrica, Slovak Republic
smutna@fpv.umb.sk

Multiplicative generator φ of a triangular norm is a special monotone function $\varphi : [0,1] \to [0,1]$ with fixed point 1, $\varphi(1) = 1$ and $\varphi(0) < 1$. The corresponding t-norm T is defined by means of φ as follows:

$$T^*(x,y) = \varphi^{(-1)}(\varphi(x).\varphi(y)),$$

where $\varphi^{(-1)} : [0,1] \to [0,1]$ is a so-called pseudo-inverse of φ defined by

$$\varphi^{(-1)}(t) = \sup\{x \in [0,1]; \varphi(x) < t\}$$

with convention $\sup \emptyset = 0$.
If strictly increasing function φ is left continuous, but non-continuous, then associativity of induces operator is violated, see in [4].
However, then the operation

$$T_*(x,y) = \begin{cases} \min(x,y), & \text{if } \max(x,y) = 1 \\ \varphi(\varphi^{(-1)}(x).\varphi^{(-1)}(y)), & \text{otherwise} \end{cases}$$

defines a left-continuous (and non-continuous) t-norm. Example is given in [3].
More, for this mention t-norm T_* there exists suitable function $F : [0,1]^2 \to [0,1]$ such that the operator $T_K : [0,1]^2 \to [0,1]$ which is defined by

$$T_K(x,y) = T_*(x,y) + F(x,y)$$

is a non-continuous t-norm. This t-norm is Archimedean with continuous diagonal.

References

[1] R.Mesiar, *Generated conjunctors and related operators in MV-logic as a basic for AI applications*, Proc. EJCAI, Brighton 1998.

[2] R.Mesiar, *Wild t-norms of G. Krause*, Abstracts of FSTA 2000, 132-133.

[3] D. Smutná, *A note on non-continuous t-norms*, BUSEFAL 76, (1998), 19-24.

[4] P.Viceník, *A note on generators of t-norms*, BUSEFAL 75, (1998), 33-38.

Research suported by grant VEGA 1/7146/20.