ON LIEB'S CONJECTURE FOR THE WEHRL ENTROPY OF BLOCH COHERENT STATES

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In the past two decades, developments in the field of coherent states and their application have been very intensive. The basic theme of this development was to connect the coherent states intimately with the dynamical group for each physical problem. For example, in the case of the Heisenberg-Weyl H_4 group , there is a one-to-one correspondance between the Glauber coherent states $|z\rangle$ and the points in the complex z plane, which is the geometrical coset space $H_4/U(1) \otimes U(1)$. For a quantum mechanical system with density matrix ρ, Hilbert space $\mathcal{H}$, we can introduce a $\langle z|\rho|z\rangle$ probability density in this complex z plane, and we can examine $S_W(\rho) = -\int dz \langle z|\rho|z\rangle \ln\langle z|\rho|z\rangle$, the Wehrl-entropy of this system. A. Wehrl conjectured and E. H. Lieb proved in 1978 that $S_W(\rho) \geq 1$ for all ρ. The lower bound on the Wehrl-entropy is related to Heisenberg's uncertainty principle and it has been speculated that S_W can be used to measure uncertainty due to both quantum and thermal fluctuations.

Lieb suggested studying the analog of Wehrl's conjecture for Bloch coherent spin states $|\Omega\rangle$. In this case the Hilbert space can be any one of the finite dimensional spin-j representation of $SU(2)$ ($\mathcal{H} = \mathbb{C}^{2j+1}$). There is one-to-one correspondence between the Bloch coherent states $|\Omega\rangle$ and the points of the sphere $S_2 = SU(2)/U(1)$. For the density matrix ρ, we can introduce the probability density $\langle\Omega|\rho|\Omega\rangle$ on the S_2, and we can examine its Wehrl-entropy $S_W(\rho)$. Lieb conjectured in 1978 that $S_W(\rho) \geq \frac{2j}{2j+1}$, but no progress has been made in the proof of the general case.

I would like to present the charm and difficulties of Lieb's conjecture and proove it in the case $j = 1$.