LARGE ON CYCLE COVERS OF GRAPHS

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A *Cycle double cover* (CDC) of a graph G is a collection of its cycles such that each edge of G lies in exactly two of the cycles. A well known conjecture of P. D. Seymour asserts that every simple bridgeless graph has a CDC [1].

Cycle double cover conjecture lies in the very heart of the graph theory. It seems that this elementary problem has a deep topological background and only partial results are known.

We now concentrate on a weighted version of this problem: A *cycle cover* of an edge-weighted graph (G, p) is a collection of cycles in G such that every edge is contained in exactly $p(e)$ cycles in the collection. A nonnegative integer valued weight vector p is called *admissible* if the total weight of any edge-cut is even, and no edge has more than half of the total weight of any edge-cut containing it. It was proved by B. Alspach, L. Goddyn, and C. Q. Zhang [2] that a graph G has a cycle cover for every admissible weight vector p if and only if G contains no subgraph homeomorphic to Petersen's graph.

We investigated an oriented version of the weighted covering of graphs with cycles: Given a graph $G = (V, E)$ and a weight function p on edges of its symmetric difference $S(G)$, we wish to find a collection of cycles in $S(G)$ (with length ≥ 3) such that every edge $e \in E(G)$ is contained in exactly $p(e)$ cycles of the collection. Such a collection we call an *oriented cycle cover* of (G, p). A nonnegative integer valued weight vector p is called admissible if p satisfies the following two conditions:

- For each edge-cut $(U, V \setminus U)$

$$\sum_{u \in U, v \in V \setminus U} p(u,v) = \sum_{u \in V \setminus U, v \in U} p(u,v)$$

- For each edge-cut $(U, V \setminus U)$ and each edge (u, v), $u \in U,\ v \in V \setminus U$

$$p(u,v) \leq \sum_{x \in V \setminus U, y \in U (x,y) \neq (v,u)} p(x,y)$$

We have shown that G has an oriented cycle cover for each admissible weight vector p if and only if G contains no subdivision of K_4 as a subgraph.

References

[1] P.D.Seymour: *Sums of circuits, in Graph Theory and Related Topics*, Academic Press, New York (1979) p.341-355.

[2] B. Alspach, L. Goddyn, C.Q. Zhang: *Graphs with the circuit cover property*, Trans. Amer. Math. Soc., Vol 344, No.1, (1994) p.131-154.