A GENERALIZATION OF A PARTIAL SUMMATION PROCESS

Ján Mačutek

Department of Probability and Statistics, Faculty of Mathematics and Physics,
Comenius University, Mlynská dolina, 842 48 Bratislava, Slovak Republic
macutek@fmph.uniba.sk

Theoretical explorations of the Bradford law and searching for law-like hypotheses in linguistics and musicology resulted in the partial summation

$P_i = \frac{c}{i} \sum_{j=i}^{\infty} P_j^*$,

$\{P_i^*\}_{i=0}^{\infty}$ being the probability mass function (pmf.) of the generating (so-called parent) and $\{P_i\}_{i=0}^{\infty}$ of the generated (descendant) distribution, c being a proper constant.

Two types of transformations of discrete random variable are presented. The first of them is a generalization of the above mentioned summation. The transformations are:

1) ${}_{R(k,l)}P_i = \frac{R(k,l)}{i-k+l} \sum_{j=i}^{\infty} P_j^*$, $i = k, k+1, \ldots$

and $\left\{{}_{R(k,l)}P_i\right\}_{i=0}^{k-1}$ are arbitrary probabilities if $k > 0$.

$\{P_i^*\}_{i=0}^{\infty}$, $\left\{{}_{R(k,l)}P_i\right\}_{i=0}^{\infty}$ are the pmf.'s of the $R(k,l)$-parent and $R(k,l)$-descendant, respectively;$R(k,l)$ is a proper function, k is a non-negative and l a positive integer.

2) ${}_{S(m,n)}P_i^* = S(m,n)\left[(i-m+1)P_{i-n+1} - (i-m+2)P_{i-n+2}\right]$,
$i = m-n+1, m-n+2, \ldots$

and $\left\{{}_{S(m,n)}P_i^*\right\}_{i=0}^{m+n-2}$ are arbitrary probabilities if $(m,n) \neq (0,1)$.

$\{P_i\}_{i=0}^{\infty}$ is a proper pmf of the $S(m,n)$-parent, $\left\{{}_{S(m,n)P_i^*}\right\}$ is the pmf. of the $S(m,n)$-descendant, $S(m,n)$ is a proper function, m is a non-negative and n a positive integer.

In both cases formulas for the normalization functions (i.e. the functions $R(k,l)$, $S(m,n)$) are derived. Relations between the probability generating functions and moments of the parent and descendent distributions are analyzed. It is shown that the $R(k,l)$- and $S(m,n)$-transformations are under certain conditions inverse, namely, if a random variable X^* is the $R(u,v)$-parent of $X_{R(u,v)}$ then $X_{R(u,v)}$ is the $S(u,v)$-parent of X^*. It is proved that the k-displaced Salvia-Bollinger distribution (i.e. $P_i^* = (-1)^{i-k}\binom{\alpha}{i-k+1}$, $i = k, k+1, \ldots; 0 < \alpha < 1$ and P_i^*=0,

$i = 0, 1, \ldots, k-1$ if k >0) is the only distribution which is invariant in regard to the $R(k,1)$-summation, i.e. the $R(k,1)$-parent and descendant are identical.