"SUBMERSIVITY" OF A SOLUTION OF THE EQUATION $y^{(n)}(t) + \alpha_1(t) \cdot y^{(n-1)}(t) + ... + \alpha_n(t) \cdot y(t) + p(t).y(t) = r(t)$

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The aim of this cotribution is to study properties of solutions of a nonlinear n^{th}-order differential equation of the form

$$y^{(n)}(t) + \alpha_1(t) \cdot y^{(n-1)}(t) + ... + \alpha_n(t) \cdot y(t) + p(t).y(t) = r(t), \qquad (1)$$

where the functions $\alpha_i(.),\ p(.)$ and $r(.)$ are continuous on $[T_0, \infty)$ all $i \in \{1, \cdots, n\}$ and satisfy the following properties: $\lim_{t\to\infty} \alpha_i(t) = 0$, for $i = 1, \cdots, n$. In addition there exist constants $r_{max}, r_{min} > 0$ such that

$$\mid p(t) \mid < r_{max}\,, \qquad 0 < r_{min} \le r(t) \le r_{max}, \qquad \text{for all } t \in [T_0, \infty).$$

The main goal is to study certain properties of the solution of the differential equation (1), which are very appropriate for exploring the Properties A or B. One can describe these properties verbally as the ability of the function not to overcome a certain level ε for a certain time interval $[t_0, t_0 + \delta]$. The function having these properties behaves as follows: from a certain $t > t_0$, it dives under a certain level of ε and keeps being under this level maximally during a time interval δ. For the sake of brevity let us call this behavior of the function "submersivity ".

Let us find a criterion of "submersivity" as the simplest possible conditions to be imposed on the left side of the equation (1).

This property is of major importance for exploring the questions about the oscillating and nonoscillating properties of a solution, and it can be directly used to prove necessary and sufficient conditions for properties A or B.

Our results do not require any super–linearity or sub–linearity hypothesis in the whole real domain. Similar problems were posed and solved by I. T. Kiguradze. Similar properties of solutions were investigated by several authors, namely for the equation of the form $u^{(n)} = f(t, u, u', \dots, u^{(n-1)})$.

Many results also have been obtained for the equation of the type $u^{(n)} + \sum p_k(t)u^{(k)} = f(t, u, \dots, u^{(n-1)})$, with a nonoscillatory left side operator. Our aim is a little different: We consider here *the case on operator the left side of the equation (1) can be oscillatory.* The new what we bring to this problem was directly the assumption on the oscillatory left side operator.