UNIQUENESS RESULT FOR A PARABOLIC SYSTEM

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We consider the following system of parabolic equations

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\begin{aligned}
u_t &= \triangle u, & v_t &= \triangle v, & x &\in \mathbb{R}^N_+, & t &> 0,\\
-\frac{\partial u}{\partial x_1} &= v^p, & -\frac{\partial v}{\partial x_1} &= u^q, & x_1 &= 0, & t &> 0,\\
u(x,0) &= u_0(x), & v(x,0) &= v_0(x), & x &\in \mathbb{R}^N_+
\end{aligned}
$$

where $\mathbb{R}^N_+ = \{(x_1, x') : x' \in \mathbb{R}^{N-1}, x_1 > 0\}$, p, q are positive numbers, and functions u_0, v_0 in the initial conditions are nonnegative and bounded.

We show that the nonnegative solution is unique if $pq \geqslant 1$. Furthermore, we also prove uniqueness of the solution for nontrivial initial condition $(u_0, v_0) \not\equiv (0,0)$ with $pq < 1$. On the other hand, we find all nontrivial nonnegative solutions if $pq < 1$ and $(u_0, v_0) \equiv (0,0)$.