EXTENSIONALITY AND CONTINUITY OF FUZZY RELATIONS

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In this paper, we investigate connection between extensionality property of fuzzy relation and its continuity. Following conclusions result from the study of a similarity relations or more generally t-equivalences and pseudo-metrics.

Let us remind that $\mathrm{E}(x,y)$ is said to be a similarity relation on a nonempty set D (E: $\mathrm{D}^2 \to [0,1]$) if is reflexive, symmetric and t-transitive (t is a t-norm) fuzzy relation on D. Further, a fuzzy relation P: $\mathrm{D}^n \to [0,1]$ is extensional w.r.t. a similarity E on D iff for each $x_1,...,x_n,y_1,...,y_n \in \mathrm{D}$, $\mathrm{E}(x_1,y_1)\,\&...\&\mathrm{E}(x_n,y_n) \leq \mathrm{P}(x_1,...,x_n) \equiv \mathrm{P}(y_1,...,y_n)$.

A fuzzy relation P is extensional w.r.t. some similarity E iff it is Lipschitz continuous w.r.t. a pseudo-metric d, where $d\,(x,y) = 1-\mathrm{E}(x,y)$, and moreover $|\mathrm{P}(x_1,...,x_n) - \mathrm{P}(y_1,...,y_n)\,| \leq \sum_{i=1}^{n} \min\left(d\left(x_i,y_i\right),1\right)$.

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