A SURVEY OF MANY-VALUED LOGICS IN COMPUTATION

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Our talk will concentrate on two-valued, three-valued and four-valued logic in the context of computational logic. It will survey these three different types of logic, and consider the interactions between them. For example, we will compare their truth values, their mathematical structures and their operators. In particular, we will consider the links between the fixed points of the T_P, Φ_P and Ψ_P operators which are determined by a program P. Previous work by Apt, Pedreschi, Hitzler and Seda has shown the relationship between the two-valued operator T_P and the three-valued operator Φ_P. Currently we are investigating the other two relationships, that is, between T_P and Ψ_P, and between Φ_P and Ψ_P.

The talk will be structured as follows:

First, we will begin with an examination of two-valued logic. We will use the Apt-van Emden-Kowalski fixed point approach. This will involve the definitions of Herbrand base, Herbrand models, sets of interpretations, etc. - concepts which will act as the foundation for the talk. We will examine the mathematical structure of two-valued logic, which will involve the notion of a complete lattice. This leads on to the definition of the single-step operator T_P and its fixed points, which play a fundamental rôle in the theory.

Second, we will motivate three-valued logic by explaining the limitations of classical two-valued semantics. These limitations emerge in computer science with the need to represent non-termination of a computer program as a truth value. This leads to the introduction of the third truth value (undefined) and its interpretation within computation. In turn, this leads to the definition of the three-valued operator Φ_P. We will find that three-valued logic has the mathematical structure of a complete semi-lattice. This has several implications that we will duly consider.

Next, four-valued logic will be motivated by the lack of a top element in the three-valued model. Thus, by extending to four truth values we can obtain greatest fixed points as well as least fixed points. This will be illustrated diagrammatically and by examples. We will consider the four-valued model offered by Nuel D. Belnap and its computational interpretation. We will examine the four-valued operator Ψ_P and the resulting mathematical structure of a complete bilattice. Now, since there are two orderings on this bilattice, this will give four operations that in turn will allow for twelve possible distributive laws. Then, given that a distributive bilattice is a pre-lattice where all these distributive laws hold, we will see that Belnap's four-valued logic is the simplest non-trivial example of a distributive bilattice.

Finally, if time permits, we will consider the interaction between the fixed points of T_P, Φ_P and Ψ_P. We will survey what is previously known and we will present the results of our research in the area.