LIFTS OF PLATONIC SOLIDS WITH VOLTAGE ASSIGNMENTS IN CYCLIC GROUPS

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Given a graph G with arc set $D(G)$ and a group Γ, a *voltage assignment* on G in Γ is any function $\alpha : D(G) \to \Gamma$ such that $\alpha(b^{-1}) = (\alpha(b))^{-1}$ for any pair of oppositely directed arcs $b, b^{-1} \in D(G)$. If $W = b_1 b_2 \ldots b_k$ is a walk in G then we set $\alpha(W) = \alpha(b_1)\alpha(b_2)\ldots\alpha(b_k)$

If Λ is a subgroup of $Aut(G)$, we say that a voltage assignment $\alpha : D(G) \to \Gamma$ is Λ-*compatible* if for each closed walk W in G and for each $A \in \Lambda$ it holds that

$$\alpha(W) = 1_\Gamma \Longleftrightarrow \alpha(AW) = 1_\Gamma.$$

It is known that Λ-compatibility is a necessary and sufficient condition for Λ to have a lift.

A map (that is, a 2-cell embedded graph in a compact surface) is said to be regular if for any two arcs a, b there is an automorphism of the map which takes a onto b. The theory of voltage graphs enables to lift a given (regular) map to new, so-called derived (regular) maps which cover the original one. In our contribution we study the non-trivial regular maps on the sphere (i.e., platonic solids) and voltage assignments on them in cyclic groups. For every platonic solid we give a complete characterisation of possible cyclic groups and voltage assignments such that the corresponding lift is again a regular map. As a sample result we state the following theorem.

Let M be the map of the icosahedron. Then there exists an $Aut(M)$-compatible voltage assignment on M in the cyclic group $\ _n$ if and only if n is a divisor of 20. All these cases correspond to regular maps which cyclically cover the map M.