16.00

1. a) $\lim\limits_{x\to 0} \frac{e^x+e^{-x}-2}{x^2} = \lim\limits_{x\to 0} \frac{e^x-e^{-x}}{2x} = \lim\limits_{x\to 0} \frac{e^x+e^{-x}}{2} = 1$

b) $\lim\limits_{x\to \infty} \left(\sqrt{x^2+4} - \sqrt{x^2-4}\right) = \lim\limits_{x\to\infty} \frac{(\sqrt{x^2+4}-\sqrt{x^2-4})(\sqrt{x^2+4}+\sqrt{x^2-4})}{\sqrt{x^2+4}+\sqrt{x^2-4}} =$

$= \lim\limits_{x\to\infty} \frac{x^2+4-(x^2-4)}{\sqrt{x^2+4}+\sqrt{x^2-4}} = \lim\limits_{x\to\infty} \frac{8}{\sqrt{x^2+4}+\sqrt{x^2-4}} = 0$

2. a) $\lim\limits_{x\to 0+} \frac{\sqrt{x}-\sqrt{0}}{x-0} = \lim\limits_{x\to 0+} \frac{\sqrt{x}}{x} = \lim\limits_{x\to 0+} \frac{1}{\sqrt{x}} = \infty$

Nie je diferencovateľná v bode 0.

b) $[\ln(x^2-x)]' = \frac{2x-1}{x^2-x}$

c) $[\ln(x^2-x)]'' = \frac{2(x^2-x)-(2x-1)^2}{(x^2-x)^2} = \frac{-2x^2+2x-1}{(x^2-x)^2}$

d) $[(e^{2x}+x)^2]' = 2(e^{2x}+x)(2e^{2x}+1) = 4e^{4x}+2e^{2x}+2x(2e^{2x}+1)$

$[(e^{2x}+x)^2]'' = 16e^{4x}+4e^{2x}+2(2e^{2x}+1)+4xe^{2x} =$

$= 16e^{4x}+8e^{2x}+4xe^{2x}+2$

3. $f(x) = \frac{\ln x}{x^2}$; t, n v bode $A = (e, ?)$

$A = (e, f(e)) = \left(e, \frac{\ln e}{e^2}\right) = \left(e, \frac{1}{e^2}\right)$

$f'(x) = \frac{\frac{1}{x}x^2 - \ln x \, 2x}{x^4} = \frac{1-2\ln x}{x^3}$, $f'(e) = \frac{1-2\ln e}{e^3} = -\frac{1}{e^3}$

$k_t = f'(e) = -\frac{1}{e^3}$, $k_n = -\frac{1}{f'(e)} = e^3$

$t: y = \frac{1}{e^2} - \frac{1}{e^3}(x-e) = -\frac{1}{e^3}x + \frac{2}{e^2}$

$n: y = \frac{1}{e^2} + e^3(x-e) = e^3x + \frac{1}{e^2} - e^4$

11.00.

1a) $\lim\limits_{x\to 0} \frac{\tan x - x}{x - \sin x} = \lim\limits_{x\to 0} \frac{\frac{1}{\cos^2 x} - 1}{1 - \cos x} = \lim\limits_{x\to 0} \frac{1-\cos^2 x}{\cos^2 x (1-\cos x)} =$

$= \lim\limits_{x\to 0} \frac{1+\cos x}{\cos^2 x} = 2$

b) $\lim\limits_{x\to\infty} x\left(\sqrt{x^2+1} - x\right) = \lim\limits_{x\to\infty} \frac{x\left(\sqrt{x^2+1}-x\right)\left(\sqrt{x^2+1}+x\right)}{\sqrt{x^2+1}+x} =$

$= \lim\limits_{x\to\infty} \frac{x(x^2+1-x^2)}{\sqrt{x^2+1}+x} = \lim\limits_{x\to\infty} \frac{x}{\sqrt{x^2+1}+x} = \lim\limits_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x^2}}+1}$

$= \frac{1}{2}$

2.a) $\lim\limits_{x\to 1^+} \frac{\sqrt{x-1} - \sqrt{0}}{x-1} = \lim\limits_{x\to 1^+} \frac{\sqrt{x-1}}{x-1} = \lim\limits_{x\to 1^+} \frac{1}{\sqrt{x-1}} = \infty$

Nie je diferencovateľná v bode 1.

b) $\left[\arctan(x-1)\right]' = \frac{1}{1+(x-1)^2}$ c) $\left[\arctan(x-1)\right]'' = -\frac{2(x-1)}{[1+(x-1)^2]^2}$

d) $\left[(e^x - x)^2\right]' = 2(e^x - x)(e^x - 1)$

e) $\left[(e^x - x)^2\right]'' = 2(e^x-1)(e^x-1) + 2(e^x - x)e^x = 2\left[2e^{2x} - (x+2)e^x + 1\right]$

3. $f(x) = \frac{3x-2}{2x-3}$; t, n v bode $A = (2, ?)$

$A = (2, f(2)) = (2, 4)$

$f'(x) = \frac{3(2x-3) - 2(3x-2)}{(2x-3)^2} = -\frac{5}{(2x-3)^2}$, $f'(2) = -5 = k_t$

$t: y = 4 - 5(x-2) = -5x + 14$

$k_n = -\frac{1}{f'(2)} = \frac{1}{5}$

$n: y = 4 + \frac{1}{5}(x-2) = \frac{1}{5}x + \frac{18}{5}$