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DITKIN'S CONDITION AND IDEALS WITH AT MOST COUNTABLE HULLS IN ALGEBRAS OF FUNCTIONS ANALYTIC IN THE UNIT DISC

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The talk is based on a joint work with Antoni Wawrzyńczyk from UAM-Iztapalapa in México City.

In [AZ] and in earlier papers [F1], and [F2] there are introduced certain classes of Banach algebras of analytic functions in the unit disc $\mathbb{D} \subset \mathbb{C}$ in which every closed ideal with the almost countable hull has a standard form (in a sense defined by the authors).

In all mentioned cases it is assumed (in a more or less explicite form) that a considered algebra $\mathcal{B}$ is embedded in the algebra $A^{(N_\mathcal{B})}(\mathbb{D})$ of functions analytic in $\mathbb{D}$ (with pointwise multiplication) and of class $C^{(N_\mathcal{B})}$ on the closed disc $\overline{\mathbb{D}}$ for some nonnegative integer $N_\mathcal{B}$. Moreover, it is assumed that the algebra $\mathcal{B}$ satisfies the *analytic Ditkin condition* which says the following: for every point z_0 in the unit circle $\mathbb{T}$ and for every function f from the algebra such that $f^{(k)}(z_0) = 0$ for $0 \leq k \leq N_\mathcal{B}$, there exists in $\mathcal{B}$ a sequence (σ_n) such that $\sigma(z_0) = 0$ for all n and $\|\sigma_n f - f\|_\mathcal{B} \to 0$ as $n \to \infty$ $\quad$ ($\|\cdot\|_\mathcal{B}$ denotes the norm of $\mathcal{B}$).

The analytic Ditkin condition is a very strong assumption which confines applicability of those results. We present a simple example of a Banach algebra $\mathcal{B}$ of analytic functions in the unit disc for which $N_\mathcal{B} = 0$ and in which Ditkin's condition does not hold. Therefore none of the results obtained in [AZ], [F1], and [F2] can be applied to describe the structure of closed ideals with at most countable hull in that algebra.

On the other hand, a form of the closed ideals with at most countable hull in that algebra is known and all such ideals are standard. The algebra $\mathcal{B}$ is only a very simple example of an algebra which contains functions having certain properties of differentiability at different boundary points of the unit disc. Algebras of this form appear in a natural way in [GMR] and [GW] as images of Gelfand transforms of convolution algebras of the Sobolev type.

We have proved that under a modified Ditkin's condition and suitably extended definition of a standard ideal the analogous result to the main theorem from [AZ] holds true.

A class of Banach algebras we are dealing with in the talk consists of subalgebras $\mathcal{B}$ of the classical disc algebra $A(\mathbb{D})$ satifying the following conditions:

(H1) The space of polynomials is a dense subset of $\mathcal{B}$.

(H2) $\lim\limits_{n\to\infty} \|\alpha^n\|_\mathcal{B}^{\frac{1}{n}} = 1$ $\qquad$ (α denotes the identity function $z \mapsto z$).

(H3) There exist $k \geq 0$ and $C > 0$ such that

$$|1 - |\lambda||^k \|f\|_\mathcal{B} \leq C\|(z-\lambda)f\|_\mathcal{B}, \quad f \in \mathcal{B},\ |\lambda| < 2.$$

(S) For every $z_0 \in \mathbb{T}$ there exists the maximal natural number $N(z_0)$ such that the functionals

$$\mathcal{B} \ni f \mapsto f^{(j)}(z_0), \quad 1 \leqslant j \leqslant N(z_0),$$

are continuous.

(D) For every $z_0 \in \mathbb{T}$ there exists a sequence (φ_n) in the algebra $\mathcal{B}$ such that $\varphi_n(z_0) = 0$ for all n and

$$\|\alpha - z_0)^{N(z_0)+1}\varphi_n \ - (\alpha - z_0)^{N(z_0)+1}\|_\mathcal{B} \to 0, \quad n \to \infty.$$

If U is an inner function and $f \in \mathcal{B}$ the symbol $U|f$ means that U divides f, i.e. there exits a function $\varphi \in H^\infty$ such that $f = U\varphi$. A closed ideal I of the algebra $\mathcal{B}$ is *standard*, according

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to our definition, if there exist an inner function U and a descending family of sets $H_n \in \mathbb{T}$ such that $N(z) \geq n$ for every $z \in H_n$ and

$$I = \{f \in \mathcal{B}\colon\ U|f \text{ and } f^{(n)}(z) = 0 \text{ for } z \in H_n \text{ and for all } n\}.$$

We obtain the following theorem.

Theorem. *If $\mathcal{B}$ is a subalgebra of the disc algebra $A(\mathbb{D})$ which satisfies conditions* (H1), (H2), (H3), (S), *and* (D), *then every closed ideal I of $\mathcal{B}$ with the at most countable hull $h(I) = \{z \in \overline{\mathbb{D}}\colon\ f(z) = 0 \text{ for } f \in I\}$ is standard.*

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