Criteria for normality via C_0-semigroups and moment sequences

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This talk is based on the paper written with I.B. Jung and J. Stochel. We are going to discuss the following two theorems.

Theorem 1. *Suppose that A is the infinitesimal generator of a C_0-semigroup $\{S(t)\}_{t\in[0,\infty)} \subseteq \boldsymbol{B}(\mathcal{H})$. Then the following conditions are equivalent*:

(i) *A is normal,*

(ii) *for every $h \in \mathcal{H}$ the functions $t \mapsto \log \|S(t)h\|$ and $t \mapsto \log \|S(t)^*h\|$ are convex on $[0,\infty)$,*

(iii) *for every $h \in \mathcal{H}$ there exists $\varepsilon_h \in (0,\infty)$ such that the functions $t \mapsto \log \|S(t)h\|$ and $t \mapsto \log \|S(t)^*h\|$ are convex on $[0,\varepsilon_h)$.*

Moreover, if A is normal, then $\mathcal{N}(S(t)) = \mathcal{N}(S(t)^*) = \{0\}$ for all $t \in [0,\infty)$.

Theorem 2. *An operator $A \in \boldsymbol{B}(\mathcal{H})$ is normal if and only if $\mathcal{N}(A) = \mathcal{N}(A^*)$ and for some integers $j,k \geq 1$ (equivalently: for all integers $j,k \geq 1$) the sequences $\{\|A^n h\|^{2j}\}_{n=0}^{\infty}$ and $\{\|A^{*n} h\|^{2k}\}_{n=0}^{\infty}$ are Hamburger moment sequences for every $h \in \mathcal{H}$.*

One-dimensional and multidimensional spectral order

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Spectral order was defined by M. P. Olson in 1971 for bounded selfadjoint operators. The motivation to consider spectral order was the fact that the set of all selfadjoint bounded operators with usual order given by quadratic forms is not lattice ordered. As shown by Kadison, the set $\mathcal{S}$ of all bounded selfadjoint operators on a complex Hilbert space $\mathcal{H}$ is an anti-lattice, which means that for $A, B \in \mathcal{S}$, a greatest lower bound for A and B exists with respect to the usual ordering "$\leqslant$" in $\mathcal{S}$ if and only if A and B are comparable (cf. [1]). A little bit earlier, Sherman proved that if the set of all selfadjoint elements of a C^*-algebra $\mathcal{A}$ of bounded linear operators on $\mathcal{H}$ is lattice ordered by "$\leqslant$", then $\mathcal{A}$ is commutative (cf. [3]). Olson showed by himself that the set of all selfadjoint elements of a von Neumann algebra of bounded linear operators on $\mathcal{H}$ is a conditionally complete lattice with respect to the spectral order (cf. [2]).

The aim of this talk is to present the spectral order in the case of unbounded selfadjoint operators and n-tuples of commuting unbounded selfadjoint operators.

References

[1] R. V. Kadison, Order properties of bounded self-adjoint operators, *Proc. Amer. Math. Soc.* **2** (1951), 505-510.

[2] M. P. Olson, The selfadjoint operators of a von Neumann algebra form a conditionally complete lattice, *Proc. Amer. Math. Soc.* **28** (1971), 537-544.

[3] S. Sherman, Order in operator algebras, *Amer. J. Math.* **73** (1951), 227-232.