Fourierova transformácia

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1 Fourierova transformácia.

$\mathcal{F}\{f(t)\} = \overset{\curlywedge}{f}(p) = \int_{-\infty}^{\infty} f(t)e^{-ipt}dt, p \in \boldsymbol{R},\ f : \boldsymbol{R} \longrightarrow \boldsymbol{C}$

$\mathcal{F}^{-1}\{f(t)\} = \overset{\curlyvee}{f}(p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{ipt}dt, p \in \boldsymbol{R},$

$\mathcal{F}\{f(t)\} = \overset{\curlywedge}{f}(p) = 2\pi\ \overset{\curlyvee}{f}(-p),$ t.j. $\overset{\curlyvee}{f}(-p) = \frac{1}{2\pi}\ \overset{\curlywedge}{f}(p).$

$L^1(\boldsymbol{R}) = \{f : \boldsymbol{R} \longrightarrow \boldsymbol{C} : \int_{-\infty}^{\infty} |f(t)|\, dt < \infty\}.$

$\mathcal{F}\left(e^{-at^2}\right) = \int_{-\infty}^{\infty} e^{-at^2}e^{-itp}dt = \sqrt{\frac{\pi}{a}}e^{-\frac{p^2}{4a}}, a > 0$

$\int_{-\infty}^{\infty} \frac{P(t)}{Q(t)}e^{-itp}dt = \frac{2\pi i}{|p|}\sum_{\left\{z\,:\,Q\left(-\frac{z}{p}\right)=0, \operatorname{Im} z>0\right\}} res_z \frac{P(-\frac{z}{p})}{Q(-\frac{z}{p})}e^{iz}$

Základná gramatika Fourierovej transformácie $(F(p) = \mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{ipt}dt, p \in \boldsymbol{R},\ )$

1. (posun v originále) $\mathcal{F}\{f(t-a)\} = e^{-ipa}\overset{\curlywedge}{f}(p) = e^{-ipa}F(p),$

2. (zmena mierky, scaling)$\mathcal{F}\{f(at)\} = \frac{1}{|a|}\overset{\curlywedge}{f}(\frac{p}{a}) = \frac{1}{|a|}F\left(\frac{p}{a}\right), a \neq 0,$

3. (pravidlo konjugácie) $\mathcal{F}\{\overline{f(-t)}\} = \overline{\overset{\curlywedge}{f}(p)} = \overline{F(p)},$

4. (posun v obraze, modulácia vzoru) $\mathcal{F}\{e^{iat}f(t)\} = \overset{\curlywedge}{f}(p-a) = F(p-a).$

5. (Obraz derivácie) Nech $f(t)$ je spojite diferencovateľná funkcia a $f,\ f' \in L^1(\boldsymbol{R}) \Rightarrow \mathcal{F}\{f'(t)\}(p) = ip\overset{\curlywedge}{f}(p).$

Ak $f, f', ..., f^{(k)}$ spojité funkce z $L^1(\boldsymbol{R}) \Rightarrow \mathcal{F}\{f^{(k)}(t)\}(p) \circeq (ip)^k\overset{\curlywedge}{f}(p)$ $\wedge\ \lim_{|p|\to\infty} p^k\overset{\curlywedge}{f}(p) = 0.$

$f' \in L^1(\mathbf{R}) \Longrightarrow \int\limits_0^\infty f'(t)dt = \lim_{t\to\infty} f(t) - f(0)$.T.j. existuje limita $\lim_{t\to\infty} f(t)$. Táto limita musí byť rovná nule, pretože $f \in L^1(\mathbf{R})$.

(Derivácia obrazu) Nech $f(t) \in L^1(\mathbf{R})$ a $tf(t) \in L^1(\mathbf{R}) \Rightarrow \mathcal{F}\{tf(t)\}(p) = i\frac{d}{dp}\overset{\curlywedge}{f}(p)$.

$f, g \in L^1(\mathbf{R})$. $\quad (f * g)(t) = \int\limits_{-\infty}^{\infty} f(s)g(t-s)ds.$

(Obraz konvolúcie) $f, g \in L^1(\mathbf{R}) \Rightarrow \overset{\curlywedge}{h}(p) = \overset{\curlywedge}{f}(p)\overset{\curlywedge}{g}(p), h = f * g.$