Vypočítajte hodnoty:

1. $\ln(-1)$ $\quad [i\pi]$

2. $\ln(-i)$ $\quad \left[-i\frac{\pi}{2}\right]$

3. $\ln(i)$ $\quad \left[i\frac{\pi}{2}\right]$

4. $\ln\left(1-\sqrt{3}i\right)$ $\quad \left[\ln 2 - i\frac{\pi}{3}\right]$

5. $\ln\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)$ $\quad \left[i\frac{\pi}{4}\right]$

6. $\ln(\mathrm{e})$ $\quad [1]$

7. $\ln(2+2i)$ $\quad \left[\ln\sqrt{8}+i\frac{\pi}{4}\right]$

8. $\ln(-2+2i)$ $\quad \left[\ln\sqrt{8}+i\frac{3\pi}{4}\right]$

9. $\ln(-2-2i)$ $\quad \left[\ln\sqrt{8}-i\frac{3\pi}{4}\right]$

10. $\ln(2-2i)$ $\quad \left[\ln\sqrt{8}-i\frac{\pi}{4}\right]$

11. $\ln(3+4i)$ $\quad \left[\ln 5 + i\operatorname{arctg}\left(\frac{4}{3}\right)\right]$

12. $\ln(-3+4i)$ $\quad \left[\ln 5 + i\left(\pi - \operatorname{arctg}\left(\frac{4}{3}\right)\right)\right]$

13. $\ln(-3-4i)$ $\quad \left[\ln 5 + i\left(\operatorname{arctg}\left(\frac{4}{3}\right) - \pi\right)\right]$

14. $\ln(3-4i)$ $\quad \left[\ln 5 - i\operatorname{arctg}\left(\frac{4}{3}\right)\right]$

15. $\ln\left(\mathrm{e}^{i\frac{\pi}{4}}\right)$ $\quad \left[i\frac{\pi}{4}\right]$

16. $\ln\left(1+\mathrm{e}^{i\frac{\pi}{3}}\right)$ $\quad \left[\ln\sqrt{3}+i\frac{\pi}{6}\right]$

17. $\mathrm{e}^{2+i\frac{\pi}{2}}$ $\quad [i\mathrm{e}^2]$

18. e^{1+i} $\quad [\mathrm{e}(\cos 1 + i\sin 1)]$

19. i^i $\quad \left[\mathrm{e}^{-\frac{\pi}{2}}\right]$

20. $i^{\frac{3}{4}}$ $\quad \left[\mathrm{e}^{i\frac{3\pi}{8}} = \cos\left(\frac{3\pi}{8}\right) + i\sin\left(\frac{3\pi}{8}\right)\right]$

21. $(-3i)^{2i}$ $\left[e^{\pi+i\ln 9} = e^{\pi}\left(\cos\left(\ln 9\right) + i\sin\left(\ln 9\right)\right)\right]$

22. i^{1+i} $\left[ie^{-\frac{\pi}{2}}\right]$

23. $(1-i)^{2+i}$ $\left[2e^{\frac{\pi}{4}}\left(\sin\left(\ln\sqrt{2}\right) - i\cos\left(\ln\sqrt{2}\right)\right)\right]$

24. $(1+i)^{\frac{1}{2}}$ $\left[\sqrt[4]{2}\left(\cos\left(\frac{\pi}{8}\right) + i\sin\left(\frac{\pi}{8}\right)\right)\right]$

25. $(1+i\sqrt{3})^{2-i}$ $\left[2e^{\frac{\pi}{3}}\Bigg(\left(\sqrt{3}\sin(\ln 2) - \cos(\ln 2)\right) + i\left(\sqrt{3}\cos(\ln 2) + \sin(\ln 2)\right)\Bigg)\right]$

26. $\sin i$ $[i\sinh 1]$

27. $\cos i$ $[\cosh 1]$

28. $\sin\left(2-3i\right)$ $[\sin 2\cosh 3 - i\cos 2\sinh 3]$

29. $\cos(1-i)$ $[\cos 1\cosh 1 + i\sin 1\sinh 1]$

30. $\cos(4+i)$ $[\cos 4\cosh 1 - i\sin 4\sinh 1]$

31. $\mathrm{tg}(2-i)$ $\left[\frac{\sin 4 - i\sinh 2}{\cosh 2+\cos 4}\right]$

32. $\mathrm{cotg}(\frac{\pi}{4} - i\ln 2)$ $\left[\frac{8}{17} + i\frac{15}{17}\right]$

33. $\arcsin(i\sinh 1)$ $[i]$

Vyjadrite reálnu a imaginárnu časť komplexnej funkcie komplexnej premennej $f(z)$, ak $z = x + iy$, kde $x \in \mathbb{R}$, $y \in \mathbb{R}$, a:

1. $f(z) = e^{z^2}$ $\left[\mathrm{Re}f(z) = u(x,y) = e^{x^2-y^2}\cos(2xy),\ \mathrm{Im}f(z) = v(x,y) = e^{x^2-y^2}\sin(2xy)\right]$

2. $f(z) = z^2\sin z$ $\left[\mathrm{Re}f(z) = u(x,y) = (x^2-y^2)\sin x\cosh y - 2xy\cos x\sinh y,\ \mathrm{Im}f(z) = v(x,y) = (x^2-y^2)\cos x\sinh y + 2xy\sin x\cosh y\right]$

3. $f(z) = \mathrm{tg}z$ $\left[\mathrm{Re}f(z) = u(x,y) = \frac{\sin 2x}{\cos 2x+\cosh 2y},\ \mathrm{Im}f(z) = v(x,y) = \frac{\sinh 2y}{\cos 2x+\cosh 2y}\right]$

Nájdite obor konvergencie mocninového radu:

$$\sum_{n=0}^{\infty}\cos(in)z^n.$$

$\left[K\left(0,\frac{1}{e}\right) = \{z\in\mathbb{C}; |z| < \frac{1}{e}\}\right]$