Pomocou Cauchyho integrálnej vety vypočítajte integrály po jednoduchých, uzavretých, po častiach hladkých, kladne orientovaných krivkách C:

1. $\int_C \frac{z+4}{z^2+2z+5}dz$; $C = \{z \in \mathbb{C}; |z| = 1\}$ $[0]$

2. $\int_C \frac{z^2+5}{z^2+1}dz$; $C = \{z \in \mathbb{C}; 4(\mathrm{Re}z)^2 + 16(\mathrm{Im}z)^2 = 1\}$ $[0]$

3. $\int_C \frac{\mathrm{e}^z+1}{z+i}dz$; $C = \{z \in \mathbb{C}; |z| = \frac{1}{2}\}$ $[0]$

4. $\int_C \frac{z+2}{z^2-2z+2}dz$; $C = \{z \in \mathbb{C}; |z+1| = 1\}$ $[0]$

Pomocou Cauchyho integrálnej vety alebo Cauchyho integrálnej formuly vypočítajte integrály po jednoduchých, uzavretých, po častiach hladkých, kladne orientovaných krivkách C:

1. $\int_C \frac{\mathrm{e}^z \cos z}{(1+z^2)\sin z}dz$; $C = \{z \in \mathbb{C}; |z-2-i| = \sqrt{2}\}$ $[0]$

2. $\int_C \frac{\sin z}{z+i}dz$; $C = \{z \in \mathbb{C}; |z-i| = 1\}$ $[0]$

3. $\int_C \frac{1}{(z-2)(z+2i)}dz$; $C = \{z \in \mathbb{C}; |z| = 1\}$ $[0]$

4. $\int_C \frac{2z^2-3z+4}{z+1}dz$; $C = \{z \in \mathbb{C}; |z| = \frac{1}{2}\}$ $[0]$

5. $\int_C \frac{2z^2-3z+4}{z+1}dz$; $C = \{z \in \mathbb{C}; |z| = \frac{3}{2}\}$ $[18\pi i]$

6. $\int_C \frac{2z^2-3z+4}{z+1}dz$; $C = \{z \in \mathbb{C}; |z+1| = 1\}$ $[18\pi i]$

7. $\int_C \frac{\sin z}{z}dz$; $C = \{z \in \mathbb{C}; |z| = 1\}$ $[0]$

8. $\int_C \frac{\cos z}{z}dz$; $C = \{z \in \mathbb{C}; |z| = 1\}$ $[2\pi i]$

9. $\int_C \frac{z^2}{z-2i}dz$; $C = \{z \in \mathbb{C}; |z| = 3\}$ $[-8\pi i]$

10. $\int_C \frac{z^2}{z-2i}dz$; $C = \{z \in \mathbb{C}; |z| = 1\}$ $[0]$

11. $\int_C \frac{\sin(\frac{\pi}{4}z)}{z^2-1}dz$; $C = \{z \in \mathbb{C}; |z-1| = 1\}$ $\left[\frac{\pi i}{\sqrt{2}}\right]$

12. $\int_C \frac{\cos z}{(z-i)^3}dz$; C je kladne orientovaný obvod štvorca s vrcholmi v bodoch 1, $1+2i$, $-1+2i$, -1 $[-i\pi \cosh 1]$

13. $\int_C \frac{\mathrm{e}^{iz}}{z^2+1}dz$; $C = \{z \in \mathbb{C}; |z-2i| = \frac{3}{2}\}$ $\left[\frac{\pi}{\mathrm{e}}\right]$

14. $\int_C \frac{z}{z^4-1}dz$; $C = \{z \in \mathbb{C}; |z-r| = r > 1, r \in \mathbb{R}\}$ $\left[\frac{\pi i}{2}\right]$

15. $\int_C \frac{e^z+1}{z+i}dz;\ C = \{z \in \mathbb{C}; |z+i| = 2\}$ $\left[2\pi \sin 1 + 2\pi i(1+\cos 1)\right]$

16. $\int_C \frac{z^2}{(z-4)(z^2+4)}dz;\ C = \{z \in \mathbb{C}; |z-1+i| = 2\}$ $\left[-\frac{2\pi}{5} + i\frac{\pi}{5}\right]$

17. $\int_C \frac{\sin z}{z^2+1}dz;\ C = \{z \in \mathbb{C}; |z+i| = 1\}$ $\left[i\pi \sinh 1\right]$

18. $\int_C \frac{z+2}{z^2-2z+2}dz;\ C = \{z \in \mathbb{C}; |z-1-2i| = 2\}$ $\left[\pi(3+i)\right]$

19. $\int_C \frac{1}{z^4-1}dz;\ C = \{z \in \mathbb{C}; |z-1-i| = \sqrt{2}\}$ $\left[\frac{\pi}{2}(-1+i)\right]$

Vypočítajte $\int_C \frac{1}{z^2-i}dz$, ak C je jednoduchá, uzavretá, po častiach hladká, kladne orientovaná krivka, na ktorej neležia korene menovateľa $z_0 = \frac{\sqrt{2}}{2}(1+i)$, $z_1 = -\frac{\sqrt{2}}{2}(1+i)$ a platí:

1. $z_0 \in \text{Int}C,\ z_1 \notin \text{Int}C$ $\left[\frac{\pi}{\sqrt{2}}(1+i)\right]$
2. $z_0 \notin \text{Int}C,\ z_1 \in \text{Int}C$ $\left[-\frac{\pi}{\sqrt{2}}(1+i)\right]$
3. $z_0 \in \text{Int}C,\ z_1 \in \text{Int}C$ $[0]$
4. $z_0 \notin \text{Int}C,\ z_1 \notin \text{Int}C$ $[0]$

Vypočítajte $\int_C \frac{1}{z^2+9}dz$, ak C je jednoduchá, uzavretá, po častiach hladká, kladne orientovaná krivka, na ktorej neležia korene menovateľa $z_0 = 3i$, $z_1 = -3i$ a platí:

1. $z_0 \in \text{Int}C,\ z_1 \notin \text{Int}C$ $\left[\frac{\pi}{3}\right]$
2. $z_0 \notin \text{Int}C,\ z_1 \in \text{Int}C$ $\left[-\frac{\pi}{3}\right]$
3. $z_0 \in \text{Int}C,\ z_1 \in \text{Int}C$ $[0]$

Vypočítajte $\int_C \frac{e^z}{z(1-z)^3}dz$, ak C je jednoduchá, uzavretá, po častiach hladká, kladne orientovaná krivka, pričom:

1. $0 \in \text{Int}C,\ 1 \in \text{Ext}C$ $[2\pi i]$
2. $0 \in \text{Ext}C,\ 1 \in \text{Int}C$ $[-i\pi e]$
3. $0 \in \text{Int}C,\ 1 \in \text{Int}C$ $[i\pi(2-e)]$