C

(1) Definícia nevlastného integrálu $\int_a^b f(x)dx$, ak a je kritický bod.

Nech
- $a, b \in R$
- $(a,b\rangle \subseteq D(f)$ 1
- $\lim_{x \to a+} f(x) = \infty$ (resp. $-\infty$) 1
- $\int_t^b f(x)dx$ existuje vlastný R-integr. pre všetky $t \in (a,b\rangle$ 1

potom ak existuje vlastná

$$\lim_{t \to a+} \int_t^b f(x)dx \overset{\text{def.}}{=} \int_a^b f(x)dx \text{ konverguje}$$

inak diverguje 2

spolu 5 bodov

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(3) Najdite $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$; na akom intervale? urobte skúšku!

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx = \int (\sin t)\,2\,dt = -2\cos t + C\Big|_{t=\sqrt{x}} =$$

subst: $\sqrt{x} = t$

1 $\frac{1}{2\sqrt{x}}dx = dt$

$\frac{dx}{\sqrt{x}} = 2dt$

$= -2\cos\sqrt{x} + C$, 3

na intervale $(0,\infty)$, 2

skúška:

$$\left[-2\cos\sqrt{x}+C\right]' = -2(-\sin\sqrt{x})\frac{1}{2\sqrt{x}}$$

1 $= \frac{\sin\sqrt{x}}{\sqrt{x}}$!

spolu 7 bodov

pokračovanie ©

(3) Plošný obsah medzi grafmi funkcií

$f(x)=|x|$, $g(x)=6-|x|$ (nakreslite)!

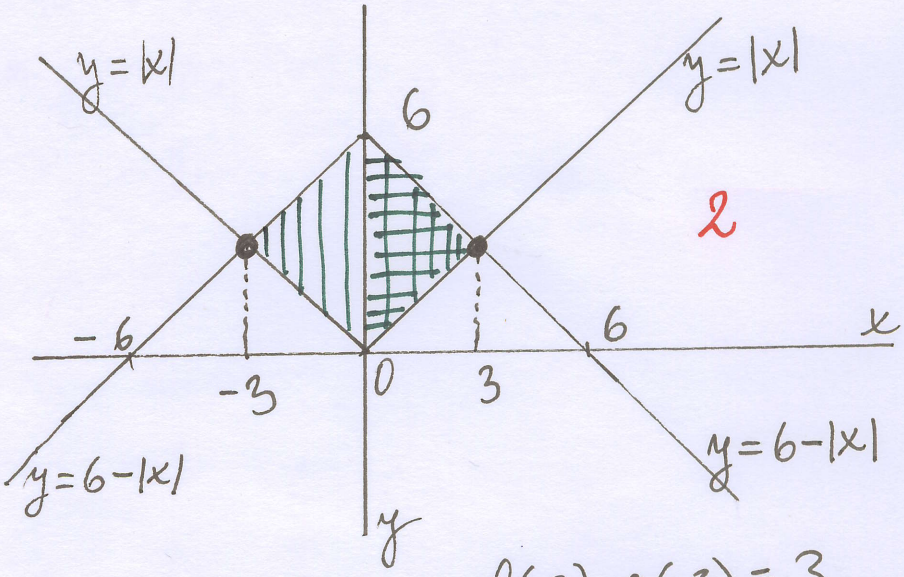
priesečníky:

$|x|=6-|x|$

$2|x|=6$ 2

$|x|=3$

$x_{1,2}=\begin{cases}3\\-3\end{cases}$



$f(-3)=g(-3)=3$

$f(3)=g(3)=3$

$$P=2\int_0^3(6-x-x)\,dx=$$

$$=2\int_0^3(6-2x)\,dx=2\left[6x-2\frac{x^2}{2}\right]_0^3=2(18-9)=18$$ 4

spolu 8 bodov

iný spôsob:

$$P=\int_{-3}^{3}(6-|x|-|x|)\,dx=\int_{-3}^{0}(6+x+x)\,dx+\int_0^3(6-x-x)\,dx$$

pretože: $x<0$ na $\langle -3,0)$, $x>0$ na $(0,3\rangle$

$$=\int_{-3}^{0}(6+2x)\,dx+\int_0^3(6-2x)\,dx=$$

$$=\left[6x+2\frac{x^2}{2}\right]_{-3}^{0}+\left[6x-2\frac{x^2}{2}\right]_0^3=$$

$$=0-\left(-18+2\frac{9}{2}\right)+(18-9-0)=36-18=18$$ 4

spolu 8 bodov