B skupina

① Kedy pre funkciu $f \subseteq R^n \times R$

(a) $\lim_{\bar{x} \to \bar{a}} f(\bar{x}) = b$ 2b

(b) f je spojitá v bode $\bar{a}$ 1b

(c) $\lim_{\bar{x} \to \bar{a} (\text{cez } A)} f(\bar{x}) = b$, kde $\bar{a}$ je hromadný bod $A \subseteq D(f)$. (definícia) 2b

1(a) Ak pre každú postupnosť bodov $\bar{a}^{(k)} \in D(f)$, $\bar{a}^{(k)} \neq \bar{a}$, $\bar{a}^{(k)} \xrightarrow[k \to \infty]{} \bar{a}$ platí $\lim_{k \to \infty} f(\bar{a}^{k}) = b \in R$ 2b

1(b) ak $\lim_{\bar{x} \to \bar{a}} f(\bar{x}) = f(\bar{a})$ (kde $O_\delta(\bar{a}) \subseteq D(f)$) 1b

1c Ak pre každú post. bodov $\bar{a}^{(k)} \in A$, $\bar{a}^{(k)} \neq \bar{a}$, $\bar{a}^{(k)} \xrightarrow[k \to \infty]{} \bar{a}$ platí $\lim_{k \to \infty} f(\bar{a}^{(k)}) = b$ 1b

$\bar{a}$ je hromadný bod A ak existuje postupnosť $\bar{a}^k \in A$, $\bar{a}^k \neq \bar{a}$, $\bar{a}^k \xrightarrow[k \to \infty]{} \bar{a}$. 1b

B ② Pre funkciu $f(x,y) = x^2 e^y + y^2$ nájdite:
spolu 10b
(a) $D(f)$, (b) stacionárne body, (c) lokálne extrémy, (d) dotykovú rovinu v bode $\bar{b} = (1,1,?)$

(a) $D(f) = R^2 = (-\infty,\infty) \times (-\infty,\infty)$ 1b

(b) (1b)
$$\begin{cases} \frac{\partial f}{\partial x} = 2x e^y & = 0 \iff x = 0 \\ \frac{\partial f}{\partial y} = x^2 e^y + 2y & = 0 \Rightarrow y = 0 \end{cases} \Bigg\} \ \bar{a} = (0,0)$$ 2b

jediný stac. bod

(c) $\frac{\partial^2 f}{\partial x^2} = 2e^y$, $\frac{\partial^2 f}{\partial y^2} = x^2 e^y + 2$, $\frac{\partial^2 f}{\partial y \partial x} = 2x e^y$ 2b

$$\left[\frac{\partial^2 f}{\partial x^2}\right]_{\substack{x=0\\y=0}} = 2, \left[\frac{\partial^2 f}{\partial y^2}\right]_{\substack{x=0\\y=0}} = 2, \left[\frac{\partial^2 f}{\partial y \partial x}\right]_{\substack{x=0\\y=0}} = 0$$

$$H_f(0,0) = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4 > 0, \left[\frac{\partial^2 f}{\partial x^2}\right]_{\substack{x=0\\y=0}} = 2 > 0$$ 2b

teda f má v $\bar{a} = (0,0)$ ostré lokálne minimum

(d) $g(x,y,z) = x^2 e^y + y^2 - z$ $\quad (z = f(x,y))$

$$\bar{b} = (1, 1, e+1)$$ 2b

$$\frac{\partial g}{\partial x} = 2x e^y, \frac{\partial g}{\partial y} = x^2 e^y + 2y, \frac{\partial g}{\partial z} = -1$$

$$\left[\frac{\partial g}{\partial x}\right]_{\substack{x=1\\y=1\\z=e+1}} = 2e, \left[\frac{\partial g}{\partial y}\right]_{\substack{x=1\\y=1\\z=e+1}} = e+2, \left[\frac{\partial g}{\partial z}\right]_{\substack{x=1\\y=1\\z=e+1}} = -1$$

$$\rho \equiv \nabla g(\bar{b}) \cdot (\bar{x} - \bar{b}) = (2e, e+2, -1)(x-1, y-1, z-e-1) = 0$$

$$\rho \equiv 2e(x-1) + (e+2)(y-1) - (z-e-1) = 0$$ 3b