

Vypočítajte limity alebo ukážte, že neexistujú:

1. $\lim_{(x,y) \rightarrow (2,3)} \frac{x}{x+y}$ $\left[\frac{2}{5}\right]$
2. $\lim_{(x,y) \rightarrow (0,0)} \frac{x}{x+y}$ $[\neq]$
3. $\lim_{(x,y) \rightarrow (2,2)} \frac{x^3 - y^3}{x^2 - y^2}$ $[3]$
4. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 + y^3}{x^2 - y^2}$ $[\neq]$
5. $\lim_{(x,y) \rightarrow (0,0)} \frac{x+y}{x-y}$ $[\neq]$
6. $\lim_{(x,y) \rightarrow (0,0)} \frac{2xy}{xy + 2x - y}$ $[\neq]$
7. $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2 + y^4}$ $[\neq]$
8. $\lim_{(x,y) \rightarrow (0,0)} \frac{x+y}{\sqrt{x^2 + y^2}}$ $[\neq]$
9. $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^3 + y}$ $[\neq]$
10. $\lim_{(x,y) \rightarrow (-2,1)} \frac{(2x+y)^2 - 9}{4xy + 2y^2 + 6y}$ $[-3]$
11. $\lim_{(x,y) \rightarrow (2,3)} \frac{y-3}{x+y-5}$ $[\neq]$
12. $\lim_{(x,y) \rightarrow (0,0)} \frac{2 - \sqrt{4-xy}}{xy}$ $\left[\frac{1}{4}\right]$
13. $\lim_{(x,y) \rightarrow (0,2)} \frac{3y^2 - 3xy - 6y}{1 - \sqrt{x-y+3}}$ $[12]$
14. $\lim_{(x,y) \rightarrow (0,0)} \frac{3(x^2 + y^2)}{\sqrt{x^2 + y^2 + 4} - 2}$ $[12]$
15. $\lim_{(x,y) \rightarrow (3,4)} \frac{4 - \sqrt{x+3y+1}}{15 - x - 3y}$ $\left[\frac{1}{8}\right]$
16. $\lim_{(x,y) \rightarrow (0,2)} \frac{\sin(xy)}{x}$ $[2]$
17. $\lim_{(x,y) \rightarrow (4,0)} \frac{\operatorname{tg}(xy)}{y}$ $[4]$
18. $\lim_{(x,y) \rightarrow (0,0)} (1 + x^2y^2)^{\frac{1}{x^2y^2}}$ $[e]$

$$19. \lim_{(x,y) \rightarrow (0,0)} (1 + x^2 y^2)^{\frac{1}{x^2 + y^2}} \quad [1]$$

$$20. \lim_{(x,y) \rightarrow (0,2)} (1 + xy)^{\frac{2}{x^2 + xy}} \quad [e^2]$$

$$21. \lim_{(x,y) \rightarrow (0,0)} \frac{e^{-\frac{1}{x^2 + y^2}}}{x^4 + y^4} \quad [0]$$

$$22. \lim_{(x,y) \rightarrow (-\sqrt{2}, \sqrt{2})} \frac{9}{\sqrt{4 - x^2 - y^2}} \quad [+ \infty]$$

$$23. \lim_{(x,y) \rightarrow (\infty, \infty)} \frac{x^2 + y^2}{e^{(x+y)}} \quad [0]$$

$$24. \lim_{(x,y) \rightarrow (0,0)} \frac{x^3 + y^3}{x^2 + y^2} \quad [0]$$

$$25. \lim_{(x,y) \rightarrow (0,0)} (x^2 + y^2) \sin\left(\frac{1}{xy}\right) \quad [0]$$

$$26. \lim_{(x,y) \rightarrow (0,0)} \frac{\sin(xy)}{\sqrt{x^2 + y^2}} \quad [0]$$

Vyšetrite spojitosť funkcie f v bode a , ak:

$$1. a = (0, 0) \text{ a } f(x, y) = \begin{cases} (xy) \frac{x^2 - y^2}{x^2 + y^2}, & \text{pre } (x, y) \neq (0, 0); \\ 0, & \text{pre } (x, y) = (0, 0). \end{cases}$$

[funkcia f je spojitá v bode a]

$$2. a = (0, 0) \text{ a } f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2}, & \text{pre } (x, y) \neq (0, 0); \\ 0, & \text{pre } (x, y) = (0, 0). \end{cases}$$

[funkcia f nie je spojitá v bode a]

$$3. a = (0, 0) \text{ a } f(x, y) = \begin{cases} \frac{\sin(x^2 + y^2)}{x^2 + y^2}, & \text{pre } (x, y) \neq (0, 0); \\ 1, & \text{pre } (x, y) = (0, 0). \end{cases}$$

[funkcia f je spojitá v bode a]

$$4. a = (0, 0) \text{ a } f(x, y) = \begin{cases} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1}, & \text{pre } (x, y) \neq (0, 0); \\ 2, & \text{pre } (x, y) = (0, 0). \end{cases}$$

[funkcia f je spojitá v bode a]

$$5. a = (0, 0) \text{ a } f(x, y) = \begin{cases} \frac{e^{-\frac{1}{x^2 + y^2}}}{x^2 + y^2}, & \text{pre } (x, y) \neq (0, 0); \\ 1, & \text{pre } (x, y) = (0, 0). \end{cases}$$

[funkcia f nie je spojitá v bode a]

Určte $k \in \mathbb{R}$ tak, aby funkcia f bola spojitá v bode a , ak:

$$a = (3, 0) \text{ a } f(x, y) = \begin{cases} \frac{\sin(6xy)}{y}, & \text{pre } y \neq 0; \\ k, & \text{pre } (x, y) = (x, 0). \end{cases}$$

$$[k = 18]$$

Dodefinujte funkciu f v bode a tak, aby v ňom bola spojitá, ak:

$$a = (2, 2) \text{ a } f(x, y) = \frac{x^3 - y^3}{x^4 - y^4}.$$

$$\left[\tilde{f}(x, y) = \begin{cases} \frac{x^3 - y^3}{x^4 - y^4}, & \text{pre } (x, y) \neq (2, 2); \\ \frac{3}{8}, & \text{pre } (x, y) = (2, 2). \end{cases} \right]$$