Pr 1.

a) $$\lim_{(x,y)\to(0,0)} \frac{x^3}{x^2+y^2} = \lim_{(x,y)\to(0,0)} x \cdot \frac{x^2}{x^2+y^2}$$

Pritom $\lim_{(x,y)\to(0,0)} x = 0$ a funkcia $0 \leq \frac{x^2}{x^2+y^2} \leq 1$ je ohraničená v $\mathring{O}_\delta(0,0)$.

Preto $$\lim_{(x,y)\to(0,0)} \frac{x^3}{x^2+y^2} = 0$$

b) $$\frac{\partial f}{\partial x}(0,0) = \lim_{x\to 0} \frac{\frac{x^3}{x^2} - 0}{x - 0} = 1 \qquad \frac{\partial f}{\partial y}(0,0) = \lim_{y\to 0} \frac{0-0}{y-0} = 0$$

c) $$\frac{\partial f}{\partial y}(x,y) = -\frac{x^3 \cdot 2y}{(x^2+y^2)^2}$$ pre $(x,y) \neq (0,0)$

d) $$\lim_{(x,y)\to(0,0)} -2\frac{yx^3}{(x^2+y^2)^2} =$$

Pri zúžení $\varphi(t) = (t,t)$ je $\lim_{t\to 0} -2\frac{t^4}{(2t^2)^2} = -\frac{2}{4} \neq \frac{\partial f}{\partial y}(0,0)$

preto funkcia $\frac{\partial f}{\partial y}$ nie je spojitá v bode $(0,0)$

(Tiež je možné ukázať, že $\lim_{(x,y)\to(0,0)} -2\frac{yx^3}{(x^2+y^2)^2}$ neexistuje a preto $\frac{\partial f}{\partial y}$ nie je spojité v bode $(0,0)$)

Pr 2. $f(x,y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$

$f'_x = 4x^3 - 4x + 4y$ $\qquad$ $4x^3 - 4x + 4y = 0$

$f'_y = 4y^3 + 4x - 4y$ $\qquad$ $\underline{4y^3 + 4x - 4y = 0}$

$4x^3 + 4y^3 = 0 \Rightarrow x^3 = -y^3$

$x = -y$

Dosadením (napr.) do prvej rovnice:

$4x^3 - 4x - 4x = 0 \Leftrightarrow x(x^2 - 2) = 0$

$x_1 = 0 \quad x_2 = \sqrt{2} \quad x_3 = -\sqrt{2}$

$y_1 = 0 \quad y_2 = -\sqrt{2} \quad y_3 = +\sqrt{2}$

Stac. body $A = [0,0]$, $B = [\sqrt{2}, -\sqrt{2}]$, $C = [-\sqrt{2}, \sqrt{2}]$

$f''_{xx} = 12x^2 - 4$ $\qquad$ $f''_{xy} = 4$

$f''_{yy} = 12y^2 - 4$

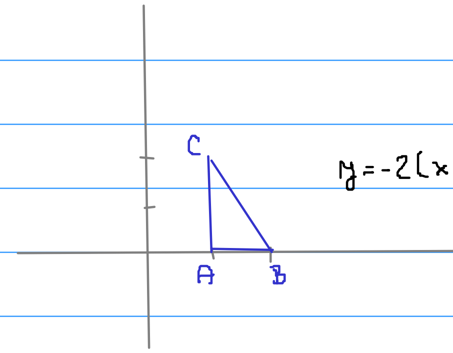
$M(A) = \begin{pmatrix} -4 & 4 \\ 4 & -4 \end{pmatrix}$ $\quad$ $d_1 = -4 < 0$, $d_2 = 0$ $\quad$ pre bod A nevieme rozhodnúť podľa Sylvestrovho kritéria

$M(B) = \begin{pmatrix} 20 & 4 \\ 4 & 20 \end{pmatrix}$ $\quad$ $d_1 = 20 > 0$, $d_2 = 400 - 16 > 0$ $\quad$ B je bod OLMIN

$M(C) = \begin{pmatrix} 20 & 4 \\ 4 & 20 \end{pmatrix}$ $\quad$ $d_1 = 20 > 0$, $d_2 = 400 - 16 > 0$ $\quad$ C je bod OLMIN

Pr 3. $\iint\limits_M \frac{1}{x+y}\,dx\,dy$

$A=[1,0]$ $B=[2,0]$ $C=[1,2]$



$y=-2(x-2)=4-2x$

$1\le x\le 2$

$0\le y\le 4-2x$

$$\int_1^2\int_0^{4-2x}\frac{1}{x+y}\,dy\,dx = \int_1^2\Big[\ln(x+y)\Big]_0^{4-2x}dx = \int_1^2 \ln(4-x)-\ln x\,dx =$$

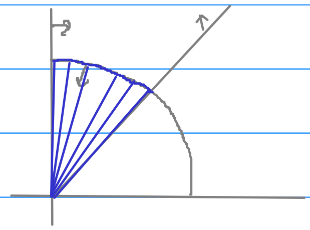
$f'=1$ $g=\ln(4-x)$ $f=x$ $g'=-\frac{1}{4-x}$ $f'=1$ $g=\ln x$ $g'=\frac{1}{x}$

$$=\Big[x\ln(4-x)\Big]_1^2-\int_1^2\frac{x}{x-4}\,dx-\Big[x\cdot\ln x\Big]_1^2+\int_1^2 x\,\frac{1}{x}\,dx=$$

$$=2\ln 2-\ln 3-\int_1^2 1+\frac{4}{x-4}\,dx-2\cdot\ln 2+0+[x]_1^2=-\ln 3-[x]_1^2-4\Big[\ln|x-4|\Big]_1^2+[x]_1^2=$$

$$=-\ln 3-4\ln 2+4\ln 3=3\ln 3-4\ln 2$$

Pr 4. $\iint_M \sqrt{x^2+y^2+2}\,dx\,dy \overset{*}{=}$ Subst.: $x = r\cos\varphi$, $y = r\sin\varphi$ $J = r$



$0 \le r \le \sqrt{2}$

$\frac{\pi}{4} \le \varphi \le \frac{\pi}{2}$

$$\overset{*}{=} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^{\sqrt{2}} \sqrt{r^2+2}\cdot r\,dr\,d\varphi = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_2^4 \sqrt{t}\cdot\frac{1}{2}\,dt\,d\varphi = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2}\cdot\left[2\cdot\frac{t^{\frac{3}{2}}}{3}\right]_2^4 d\varphi = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{4^{\frac{3}{2}}-2^{\frac{3}{2}}}{3}\,d\varphi =$$

$t = r^2+2$

$dt = 2r\,dr$

$$= \frac{8-\sqrt{8}}{3}\cdot[\varphi]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \pi\cdot\frac{8-\sqrt{8}}{3\cdot 4} = \pi\,\frac{4-\sqrt{2}}{6}$$