

5 FOURIEROVE RADY

Rozviňte do Fourierovho radu funkciu

1. $f(t) = t, \quad t \in [0, 1]$
2. $f(t) = t, \quad t \in [-1, 1]$
3. $f(t) = 1 - t, \quad t \in [0, 2]$
4. $f(t) = 1 - t, \quad t \in [0, \pi]$
5. $f(t) = t^2, \quad t \in [-1, 1]$
6. $f(t) = (t + 1)^2, \quad t \in [-1, 1]$
7. $f(t) = \sin t, \quad t \in [0, 1]$
8. $f(t) = \cosh t, \quad t \in [-\pi, \pi]$

Rozviňte do sinusového a do kosinusového radu funkciu

1. $f(t) = t, \quad t \in [0, 1]$
2. $f(t) = t^2, \quad t \in [0, 1]$
3. $f(t) = 1 - (1 - t)^2, \quad t \in [0, 2]$
4. $f(t) = \cos 2t, \quad t \in [0, \pi]$
5. $f(t) = e^t, \quad t \in [0, \pi]$
6. $f(t) = \begin{cases} 1, & t \in [0, 1] \\ 2, & t \in [1, 3] \end{cases}$
7. $f(t) = \begin{cases} 1, & t \in [0, 1] \\ 2 - t, & t \in [1, 2] \end{cases}$
8. $f(t) = \begin{cases} t, & t \in [0, 1] \\ \frac{3-t}{2}, & t \in [1, 3] \end{cases}$
9. $f(t) = t - t^3, \quad t \in [0, 1]$ Porovnajzte rýchlosť konvergenzie sínusového a kosínusového radu. Vypočítajte súčet kosínusového radu pre $t = 0$.
10. $f(t) = t, \quad t \in [0, 1]$ (viď 1.) Výsledky prepíšte do amplitúdového tvaru a nakreslite amplitúdové spektrum.

Výsledky.

1. $\frac{1}{2} + \sum_{k=1}^{\infty} \frac{(-1)^k}{k\pi} \sin 2k\pi t$
2. $\sum_{k=1}^{\infty} \frac{(-2)(-1)^k}{k\pi} \sin k\pi t$
3. $\sum_{k=1}^{\infty} \frac{(2)^k}{k\pi} \sin k\pi t$
4. $1 - \frac{\pi}{2} + \sum_{k=1}^{\infty} \frac{(1)^k}{k} \sin 2kt$
5. $\frac{1}{3} + \sum_{k=1}^{\infty} \frac{4(-1)^k}{k^2\pi^2} \cos k\pi t$
6. $\frac{4}{3} + 4 \left(\sum_{k=1}^{\infty} \frac{(-1)^k}{(k\pi)^2} \cos(k\pi t) + \frac{(-1)^{k+1}}{k\pi} \sin(k\pi t) \right)$
7. $1 - \cos(1) + \sum_{k=1}^{\infty} \left(\frac{2-4k\pi \cos(1)}{1-(2k\pi)^2} \cos(2k\pi t) - \frac{4k\pi \sin(1)}{1-(2k\pi)^2} \sin(2k\pi t) \right)$

$$8. \frac{\sinh \pi}{\pi} + \sum_{k=1}^{\infty} \sinh(\pi) \frac{2(-1)^k}{\pi(1-k^2)} \cos(kt)$$

$$1. \text{ a, } \frac{1}{2} + \sum_{k=1}^{\infty} \frac{2(-1 + (-1)^k)}{k^2 \pi^2} \cos k\pi t$$

$$\text{b, } \sum_{k=1}^{\infty} \frac{-2(-1)^k}{k\pi} \sin k\pi t$$

$$2. \text{ a, } \frac{1}{3} + \sum_{k=1}^{\infty} \frac{2(-1)^k}{k^2 \pi^2} \cos k\pi t$$

$$\text{b, } \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{k\pi} \sin k\pi t$$

$$3. \text{ a, } \frac{8}{3} + \sum_{k=1}^{\infty} \frac{(-8)(1 + (-1)^k)}{k^2 \pi^2} \cos \frac{k\pi}{2} t$$

$$\text{b, } \sum_{k=1}^{\infty} \frac{16(1 + (-1)^k)}{k^3 \pi^3} \sin \frac{k\pi}{2} t$$

$$5. \text{ a, } \frac{1}{\pi}(e^{\pi} - 1) + \sum_{k=1}^{\infty} \frac{2}{\pi} \frac{(-1)^k e^{\pi} - 1}{1 + k^2} \cos kt$$

$$\text{b, } \sum_{k=1}^{\infty} \frac{2}{\pi} \frac{(-1)^k e^{\pi} + 1}{1 + k^2} \sin kt$$

$$6. \text{ a, } \frac{5}{3} + \sum_{k=1}^{\infty} \frac{(-2)(\sin k \frac{\pi}{3})}{k\pi} \cos k \frac{\pi}{3} t$$

$$\text{b, } \sum_{k=1}^{\infty} \frac{-4(-1)^k + 2 + 2 \cos(k \frac{\pi}{3})}{k\pi} \sin k \frac{\pi}{3} t$$

$$7. \text{ a, } \frac{3}{4} + \sum_{k=1}^{\infty} \frac{-4}{k^2 \pi^2} ((-1)^k - \cos k \frac{\pi}{2}) \cos k \frac{\pi}{2} t$$

$$\text{b, } \sum_{k=1}^{\infty} \frac{2k\pi + 4 \sin k \frac{\pi}{2}}{k^2 \pi^2} \sin k \frac{\pi}{2} t$$

$$8. \text{ a, } \frac{1}{2} + \sum_{k=1}^{\infty} 3 \frac{3 \cos k \frac{\pi}{3} + (-1)^{k+1} - 2}{k^2 \pi^2} \cos \frac{k\pi}{3} t$$

$$\text{b, } \sum_{k=1}^{\infty} \frac{9 \sin k \frac{\pi}{3}}{k^2 \pi^2} \sin \frac{k\pi}{3} t$$

$$9. \text{ a, } \frac{1}{4} + \sum_{k=1}^{\infty} \frac{1}{k^2 \pi^2} [2(-1)^{k+1} + \frac{12(-1)^k - 12}{k^2 \pi^2}] \cos k\pi t$$

$$\text{b, } \sum_{k=1}^{\infty} \frac{12(-1)^{k+1}}{k^3 \pi^3} \sin k\pi t$$