3. príklad   Viazané extrémy

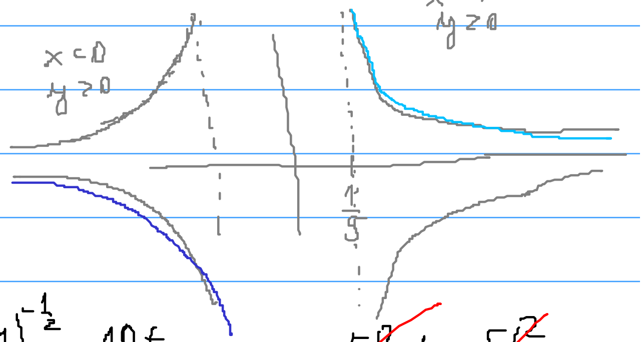
$f(x,y) = 2x + \frac{1}{4}y$ $\frac{1}{x^2}+\frac{1}{y^2}=5$ $g(x,y) = \frac{1}{x^2}+\frac{1}{y^2}-5=0$

$D_f = \mathbb{R}^2$ $y^2 = \frac{1}{5-\frac{1}{x^2}}$ $x \neq 0 \wedge y \neq 0$

Ak $y>0$ $y = \sqrt{\frac{1}{5-\frac{1}{x^2}}} = \sqrt{\frac{x^2}{5x^2-1}}$

Ak $x>0$ $y = \frac{x}{\sqrt{5x^2-1}}$

Ak $x<0$ $y = \frac{-x}{\sqrt{5x^2-1}}$



$\varphi(t) = \left(t, \frac{t}{\sqrt{5t^2-1}}\right)$

$h(t) = f(\varphi(t)) = 2t + \frac{1}{4}\frac{t}{\sqrt{5t^2-1}}$

$h'(t) = 2 + \frac{1}{4}\,\frac{\sqrt{5t^2-1} - t\cdot\frac{1}{2}(5t^2-1)^{-\frac{1}{2}}\cdot 10t}{5t^2-1} = 2 + \frac{1}{4}\,\frac{\cancel{5t^2}-1-\cancel{5t^2}}{(5t^2-1)^{\frac{3}{2}}} =$

$= 2 - \frac{1}{4}\,\frac{1}{(5t^2-1)^{\frac{3}{2}}} = 0 \quad \Big| \cdot 4$

$8 = \frac{1}{(5t^2-1)^{\frac{3}{2}}}$

$(5t^2-1)^{\frac{3}{2}} = \frac{1}{8}$

$5t^2-1 = \frac{1}{4}$ $5t^2 = \frac{5}{4}$ $t^2 = \frac{1}{4}$ $t = \frac{1}{2}$

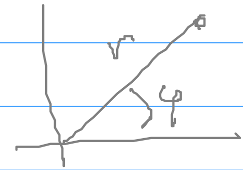
St bod $t_0 = \frac{1}{2}$ $A = \left[\frac{1}{2}, \frac{\frac{1}{2}}{\sqrt{\frac{1}{4}}}\right] = \left[\frac{1}{2}, 1\right]$

$-$ ↘ $\frac{1}{2}$ ↗ $+$

bod viazaného OLMIN

$C = \left[-\frac{1}{2}, -1\right]$ bod VOLMAX

Pr 5. Vypočítajte (pomocou polárnych súradníc)



$$\iint_M xy \, dx\, dy \overset{*}{=}$$

$x = r\cos\varphi$

$y = r\sin\varphi$

$J = r$

$M:\ 0 \le y$

$(x-1)^2 + y^2 \le 1$

oblasť typu φr

$$\boxed{\begin{aligned} 0 \le \varphi &\le \frac{\pi}{2} \\ 0 \le r &\le 2\cos\varphi \end{aligned}}$$

$$x^2 - 2x + \cancel{1} + y^2 = \cancel{1}$$

$$\underline{r^2\cos^2\varphi} - 2r\cos\varphi + \underline{r^2\sin^2\varphi} = 0$$

$$r^2 = 2r\cos\varphi \qquad r_1 = 0$$

$$r = 2\cos\varphi$$

$$= \int_0^{\frac{\pi}{2}} \int_0^{2\cos\varphi} r^2\cos\varphi\sin\varphi \cdot r \, dr \, d\varphi = \int_0^{\frac{\pi}{2}} \cos\varphi\sin\varphi \left[\frac{r^4}{4}\right]_0^{2\cos\varphi} d\varphi =$$

$$= \int_0^{\frac{\pi}{2}} \cos\varphi\sin\varphi \, 4\cos^4\varphi \, d\varphi =$$

$\varphi = \frac{\pi}{2} \quad y = \cos\frac{\pi}{2} = 0$

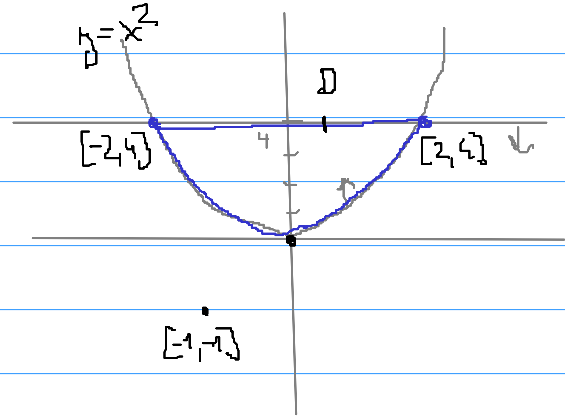
$y = \cos\varphi \qquad dy = -\sin\varphi \, d\varphi \qquad \varphi = 0 \quad y = \cos 0 = 1$

$$= -\int_1^0 4y^5\,dy = 4\int_0^1 y^5\,dy = 4\left[\frac{y^6}{6}\right]_0^1 = \frac{4}{6}$$

3 Pr. Nájdite absolútne extrémy funkcie

$$f(x,y) = 2x^3 + 4x^2 + y^2 - 2xy$$

na množine M : $x^2 \leq y \leq 4$



$x^2 = y \qquad y = 4$

$x_{1,2} = \pm 2$

(i) lokálne extrémy

$$f'_x = 6x^2 + 8x - 2y \stackrel{!}{=} 0$$
$$f'_y = 2y - 2x \stackrel{!}{=} 0$$

$y = x$

$6x^2 + 8x - 2x = 0$

$x^2 + x = 0$

$x_1 = 0 \qquad y_1 = 0$

$\cancel{x_2 = -1} \qquad \cancel{y_2 = -1}$ nie je z M

$A = [-2,4]$

$B = [2,4]$

$C = [0,0]$

(ii) viazané extrémy

$$f(x,y) = 2x^3 + 4x^2 + y^2 - 2xy$$

pre väzbu

$y = 4$

$$h(x) = f(x,4) = 2x^3 + 4x^2 + 16 - 8x$$

$$h'(x) = 6x^2 + 8x - 8 \overset{!}{=} 0 \qquad 3x^2 + 4x - 4 = 0 \qquad x_{1,2} = \frac{-4 \pm \sqrt{16 + 4\cdot 3\cdot 4}}{6} = \frac{-4 \pm 2\cdot 4}{6} \Big\langle \begin{matrix} \frac{2}{3} \\ -2 \end{matrix}$$

St. body: $D = [\frac{2}{3}, 4]$ $\quad E = [-2, 4] = A$

$$f(x,y) = 2x^3 + 4x^2 + y^2 - 2xy$$

pre väzbu

$y = x^2$ $\quad x = t$, $y = t^2$

$$h(t) = f(t, t^2) = \cancel{2t^3} + 4t^2 + t^4 - \cancel{2t\cdot t^2} = t^4 + 4t^2$$

$$h'(t) = 4t^3 + 8t = 0 \qquad t_1 = 0 \qquad [0,0] = C$$

$$4t(t^2+2) = 0$$

$t^2 + 2 \neq 0$

$B = [2,4]$

$$f(A) = \cancel{-16} + \cancel{4}\cdot 4 + 4^2 + 2\cdot 2\cdot 4 = 32 > \frac{22\cdot 16}{27}$$

$$f(B) = 16 + 16 + \cancel{16} - \cancel{2\cdot 2\cdot 4} = 32$$

$$f(C) = 0$$

$$\cancel{f(D)} = 2\cdot\frac{8}{27} + 4\cdot\frac{4}{9} + 16 - 2\cdot\frac{2}{3}\cdot 4 = \frac{16}{27} + \frac{48}{27} + 16 - \frac{16\cdot 9}{3\cdot 9} = 16 - \frac{5\cdot 16}{27} = \frac{27\cdot 16 - 5\cdot 16}{27} = \frac{22\cdot 16}{27}$$

Abs maximum je 32 v bodoch A, B.

Abs minimum je 0 v bode C