Pr 1. $f(x,y) = \sqrt[3]{x \cdot y^2} = (x y^2)^{\frac{1}{3}} = x^{\frac{1}{3}} \cdot y^{\frac{2}{3}}$

a) Dotyková rovina v bode $a = [2,1]$

$$z - z_0 = \frac{\partial f}{\partial x}(a)(x - x_0) + \frac{\partial f}{\partial y}(a)(y - y_0) \qquad z_0 = f(x_0, y_0) \qquad (x_0, y_0) = \bar{a}$$

$$z_0 = f(2,1) = \sqrt[3]{2 \cdot 1} = \sqrt[3]{2}$$

$$\frac{\partial f}{\partial x}(x,y) = \frac{1}{3} x^{-\frac{2}{3}} \cdot y^{\frac{2}{3}} = \frac{1}{3}\left(\frac{y}{x}\right)^{\frac{2}{3}} \qquad \frac{\partial f}{\partial x}(a) = \frac{1}{3}\left(\frac{1}{2}\right)^{\frac{2}{3}}$$

$$\frac{\partial f}{\partial y}(x,y) = x^{\frac{1}{3}} \cdot \frac{2}{3} y^{-\frac{1}{3}} = \frac{2}{3}\left(\frac{x}{y}\right)^{\frac{1}{3}} \qquad \frac{\partial f}{\partial y}(a) = \frac{2}{3}\, 2^{\frac{1}{3}}$$

$$z - \sqrt[3]{2} = \frac{1}{3}\left(\frac{1}{2}\right)^{\frac{2}{3}}(x - 2) + \frac{2}{3}\, 2^{\frac{1}{3}} \cdot (y - 1)$$

$f(x,y) = \sqrt[3]{x y^2}$

b) $\frac{\partial f}{\partial y}(0,0)$ Nedá sa dosadiť do výsledku z časti a)

Z definície $\frac{\partial f}{\partial y}(0,0) = \lim_{y \to 0} \frac{f(0,y) - f(0,0)}{y - 0} = \lim_{y \to 0} \frac{0 - 0}{y - 0} = 0$

c) $\frac{\partial f}{\partial y}(1,0)$ $\quad \frac{\partial f}{\partial y}(1,0) = \lim_{y \to 0} \frac{f(1,y) - f(1,0)}{y - 0} = \lim_{y \to 0} \frac{\sqrt[3]{y^2} - 0}{y} = \lim_{y \to 0} y^{-\frac{1}{3}} = \lim_{y \to 0} \frac{1}{\sqrt[3]{y}}$ – neex.

Pr2. $f(x,y) = xy(4x+2y+1) =$ Lok. extrémy

$= 4x^2y + 2xy^2 + xy$

St. body $f'_x = 8xy + 2y^2 + y \overset{!}{=} 0$ $y(8x+2y+1) = 0$

$f'_y = 4x^2 + 4xy + x \overset{!}{=} 0$ $x(4x+4y+1) = 0$

$\underline{\mathrm{I}}$ $x=0$ $y=0$ $A=[0,0]$

$\underline{\mathrm{II}}$ $y=0$ $x\neq 0$

$4x + 4\cdot 0 + 1 = 0$ $x = -\frac{1}{4}$ $B=[-\frac{1}{4}, 0]$

$\underline{\mathrm{III}}$ $x=0$ $y\neq 0$

$8\cdot 0 + 2y + 1 = 0$ $y = -\frac{1}{2}$ $C=[0, -\frac{1}{2}]$

$\underline{\mathrm{IV}}$ $x\neq 0$ $y\neq 0$

$8x + 2y + 1 = 0$

$4x + 4y + 1 = 0$ $\quad | \cdot (-2)$

$-8x - 8y - 2 = 0$

$x = -y - \frac{1}{4} = \frac{1}{6} - \frac{1}{4} = \frac{-2}{24} = -\frac{1}{12}$

$-6y - 1 = 0$ $y = -\frac{1}{6}$

$D = [-\frac{1}{12}, -\frac{1}{6}]$

Sylvestrovo kritérium

$f'_x = 8xy + 2y^2 + y$

$f'_y = 4x^2 + 4xy + x$

$f''_{xx} = 8y$ $\qquad f''_{xy} = 8x + 4y + 1$

$f''_{yy} = 4x$

$B = [-\frac{1}{4}, 0]$

$C = [0, -\frac{1}{2}]$

I. $M(A) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\qquad d_2 = -1 < 0$ $\qquad$ A – sedlový bod

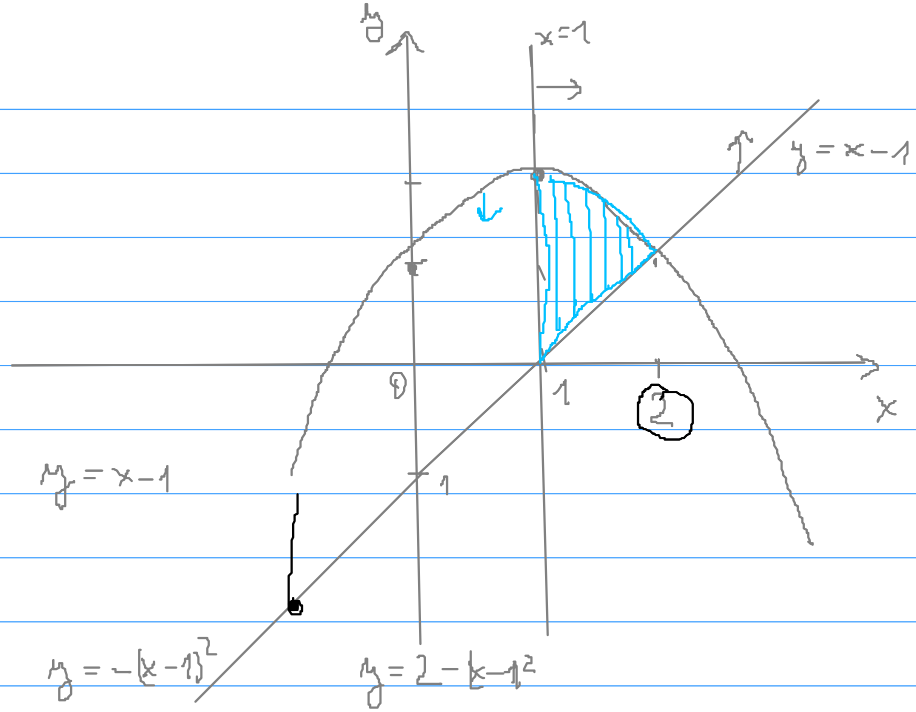
II. $M(B) = \begin{pmatrix} 0 & -1 \\ -1 & -1 \end{pmatrix}$ $\qquad d_2 = -1 < 0$ $\qquad$ B – sedlový bod

III. $M(C) = \begin{pmatrix} -4 & -1 \\ -1 & 0 \end{pmatrix}$ $\qquad d_2 = -1 < 0$ $\qquad$ C – sedlový bod

IV. $M(D) = \begin{pmatrix} -\frac{8}{6} & -\frac{8}{12} - \frac{4}{6} + 1 \\ & -\frac{4}{12} \end{pmatrix} = \begin{pmatrix} -\frac{4}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} \end{pmatrix}$

$\left.\begin{array}{l} d_2 = \frac{4}{9} - \frac{1}{9} > 0 \\ d_1 = -\frac{4}{3} < 0 \end{array}\right\}$ D – bod OLMAX

Pr3 $\displaystyle\iint_M e^x \,dx\,dy \overset{*}{=}$



$M:\quad x \geq 1$

$\qquad y \geq x-1$

$\qquad y \leq 2-(x-1)^2$

$y = x-1$ $\qquad y = 2-(x-1)^2$

$x-1 = 2-(x-1)^2 = 2-(x^2-2x+1)$

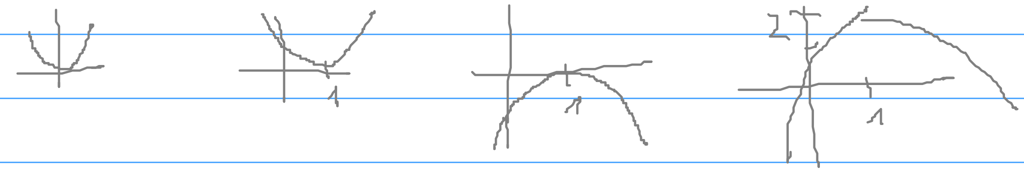
$x-1 = 2-x^2+2x-1$

$0 = 2-x^2+x$

$x_{1,2} = \dfrac{-1\pm\sqrt{1+8}}{-2} =$

$= \dfrac{-1\pm 3}{-2} = \begin{cases} 2 \\ -1 \end{cases}$

$y = x^2$ $\qquad y = (x-1)^2$ $\qquad y = -(x-1)^2$ $\qquad y = 2-(x-1)^2$



$\int 4\,dy = 4y$

$\int e^x\,dy = e^x y$

M je elem. oblasť typu xy $\qquad 1 \leq x \leq 2$

$x-1 \leq y \leq 2-(x-1)^2 = -x^2+2x+1$

$\overset{*}{=} \displaystyle\int_1^2 \left(\int_{x-1}^{-x^2+2x+1} e^x \,dy \right) dx = \int_1^2 e^x \left[y \right]_{x-1}^{-x^2+2x+1} dx = \int_1^2 e^x\left(-x^2+2x+1-x+1\right) dx =$

$= \displaystyle\int_1^2 e^x\left(-x^2+x+2\right) dx = \left[e^x\left(-x^2+x+2\right)\right]_1^2 - \int_1^2 e^x\left(-2x+1\right) dx = e^2\cdot 0 - e\cdot 3 - \left(\left[e^x(-2x+1)\right]_1^2 + 2\int_1^2 e^x dx\right) =$

$f' = e^x$ $\quad f = e^x$ $\quad f' = e^x$ $\quad f = e^x$

$g = -x^2 + x + 2$ $\quad g' = -2x + 1$ $\quad g = (-2x+1)$ $\quad g' = -2$

$$= -3e - \left(e^2(-3) + e + 2\left[e^x\right]_1^2 \right) = -3e + 3e^2 - e - 2(e^2 - e) = \underline{\underline{e^2 - 2e}}$$