Pr1. $f(x) = \frac{x-1}{x^2}$ Monotónnosť, lokálne ext., konvexnosť, inf. body

$D_f = R - \{0\}$

$f'(x) = \frac{1 \cdot x^2 - (x-1)2x}{x^4} = \frac{x^2 - 2x^2 + 2x}{x^4} = \frac{-x^2+2x}{x^4} = \frac{2-x}{x^3}$

S. Bod: $f'(x) = 0$
$2 - x = 0$
$x_0 = 2$

f'	$-$	0	$+$	2	$-$
f	$\searrow$		$\nearrow$		$\searrow$

$f'(-1) = \frac{3}{(-1)^3} = -3 < 0$

$f'(1) = \frac{1}{1} = 1 > 0$

$f'(3) = \frac{-1}{27} < 0$

f je rastúca na $(0, 2\rangle$
klesajúca na $(-\infty, 0)$ a na $\langle 2, \infty)$

Bod $x_0 = 2$ je bod OLMAX.

$f''(x) = \frac{(-1) \cdot x^3 - (2-x) \cdot 3x^2}{x^6} = \frac{-x^3 - 6x^2 + 3x^3}{x^6} = \frac{-\cancel{x^2} \cdot x - 6\cancel{x^2} + 3\cancel{x^2} \cdot x}{x^4 \cdot \cancel{x^2}} =$

$(x^3)^2 = x^3 \cdot x^3 = x^6$
$x^3 \cdot x^2 = x^5$

$= \frac{2x - 6}{x^4}$

$f''(x) = 0 \iff 2x - 6 = 0$
$x_1 = 3$

$f''(-1) = \frac{-8}{1} < 0$

$f''(1) = \frac{-4}{1} < 0$

$f''(4) = \frac{2}{4^4} > 0$

f''	$-$	0	$-$	3	$+$
f	$\cap$	$\emptyset$	$\cap$		$\cup$

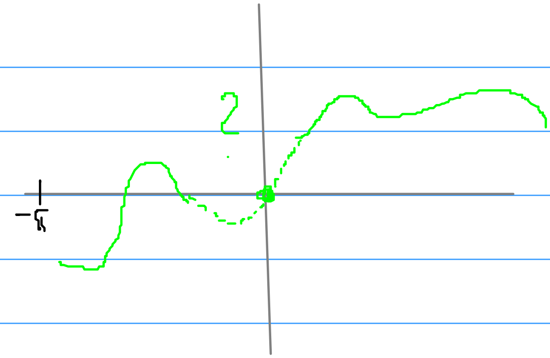
f je konvexná na $\langle 3, \infty)$

konkávna na $(-\infty, 0)$ a na $(0, 3\rangle$

bod $x_1 = 3$ je inflexný bod.

Pr 2.

$$f(x)=\begin{cases}\sqrt{\frac{1}{x}+1}-\sqrt{\frac{1}{x}} & \text{pre } x>0\\ 0 & \text{ak } x=0\\ \dfrac{\sqrt{4-x}-2}{\sin x}+\dfrac{1}{4} & \text{ak } -\pi<x<0\end{cases}$$



$$\lim_{x\to 0+} f(x)=\lim_{x\to 0+}\sqrt{\frac{1}{x}+1}-\sqrt{\frac{1}{x}}=\lim_{x\to 0+}\frac{\left(\sqrt{\frac{1}{x}+1}-\sqrt{\frac{1}{x}}\right)\left(\sqrt{\frac{1}{x}+1}+\sqrt{\frac{1}{x}}\right)}{\sqrt{\frac{1}{x}+1}+\sqrt{\frac{1}{x}}}=\lim_{x\to 0+}\frac{\cancel{\frac{1}{x}}+1-\cancel{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}+\sqrt{\frac{1}{x}}}=0$$

$$\lim_{x\to 0-} f(x)=\lim_{x\to 0-}\frac{\sqrt{4-x}-2}{\sin x}+\frac{1}{4}=\frac{1}{4}+\lim_{x\to 0-}\frac{\sqrt{4-x}-2}{\sin x}\cdot\frac{\sqrt{4-x}+2}{\sqrt{4-x}+2}=\frac{1}{4}+\lim_{x\to 0-}\frac{\cancel{4}-x-\cancel{4}}{\sin x\,(\sqrt{4-x}+2)}=$$

$$=\frac{1}{4}+(-1)\cdot\frac{1}{4}=0$$

$$\lim_{x\to 0+} f(x)=\lim_{x\to 0-} f(x)=f(0)\quad\Rightarrow\quad f \text{ je spojitá v bode } 0$$

$$\frac{st\ 1}{st\ 2} \Rightarrow \sim \sum \frac{1}{n} \text{ - diverguje}$$

Pr 3. a.)
$$\sum_{n=1}^{\infty} \frac{n+1}{n^2+n-2}$$

$$\lim_{n\to\infty} \frac{\frac{n+1}{n^2+n-2}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^2+n}{n^2+n-2} = \frac{1}{1} \qquad 0<1<+\infty \Rightarrow$$

$\Rightarrow$ Rady sa chovajú rovnako $\wedge$ $\sum_{n=1}^{\infty} \frac{1}{n}$ diverguje

$\Rightarrow$ Rad $\sum_{n=1}^{\infty} \frac{n+1}{n^2+n-2}$ tiež diverguje.

b.) Súčet
$$\sum_{n=2}^{\infty} \frac{5\cdot 2^n + (-1)^n}{3^{2n}} =$$

$$= \sum_{n=2}^{\infty} 5\cdot\left(\frac{2}{9}\right)^n + \sum_{n=2}^{\infty} \left(\frac{-1}{9}\right)^n =$$

$$\sum_{n=0}^{\infty} a q^n = a \frac{1}{1-q}$$

$a_I = 5\,\frac{4}{81}$ $\quad q_I = \frac{2}{9}$ $\qquad a_{II} = \frac{1}{81}$ $\quad q = -\frac{1}{9}$

$$= \frac{20}{81} \cdot \frac{1}{1-\frac{2}{5}} + \frac{1}{81} \cdot \frac{1}{1+\frac{1}{5}} = \frac{20}{81} \frac{9}{7} + \frac{1}{81} \frac{9}{10} =$$

Pr 4. $\int \frac{x^3-x^2+3x+4}{x^3+4x}\,dx = \int 1 - \frac{x^2+x-4}{x^3+4x}\,dx = x - \int \frac{x^2+x-4}{x(x^2+4)}\,dx \overset{*}{=}$

Del.: $(x^3-x^2+3x+4):(x^3+4x) = 1 - \frac{x^2+x-4}{x^3+4x}$

$-(x^3+4x)$

$-x^2-x+4$

$x^3+4x = x(x^2+4)$

Rozklad $\frac{x^2+x-4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$

$x^2+x-4 = A(x^2+4) + (Bx+C)\cdot x$

$x^2+x-4 = Ax^2+4A+Bx^2+Cx$

$1 = A+B$

$1 = C$

$-4 = 4A$

$B=2$

$C=1$

$A=-1$

$x - \int \frac{-1}{x} + \frac{2x+1}{x^2+4}\,dx = x + \ln|x| - \int \frac{2x}{x^2+4}\,dx - \int \frac{1}{x^2+4}\,dx =$

$= x + \ln|x| - \ln|x^2+4| - \frac{1}{2}\operatorname{arctg}\frac{x}{2} + C$

Pr 5. $\int_0^{\frac{\pi}{4}} x \cdot \operatorname{arctg} x \, dx =$

Per partes $\quad f' = x \qquad f = \frac{x^2}{2}$

$g = \operatorname{arctg} x \qquad g' = \frac{1}{1+x^2}$

$$= \left[\frac{x^2}{2} \operatorname{arctg} x \right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \frac{x^2}{2} \frac{1}{1+x^2} dx = \frac{\pi^2}{32} \cdot \operatorname{arctg} \frac{\pi}{4} - 0 - \frac{1}{2} \int_0^{\frac{\pi}{4}} \frac{x^2+1-1}{x^2+1} dx =$$

$$= \frac{\pi^2}{32} \cdot 1 - \frac{1}{2} \int_0^{\frac{\pi}{4}} 1 - \frac{1}{x^2+1} dx = \frac{\pi^2}{32} - \frac{1}{2} [x]_0^{\frac{\pi}{4}} + \frac{1}{2} [\operatorname{arctg} x]_0^{\frac{\pi}{4}} = \underline{\underline{\frac{\pi^2}{32} - \frac{1}{2} \frac{\pi}{4} + \frac{1}{2} \operatorname{arctg} \frac{\pi}{4}}}$$