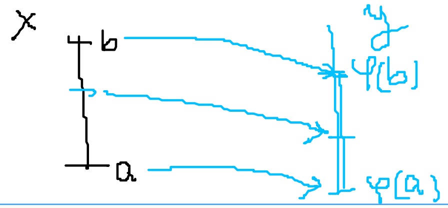
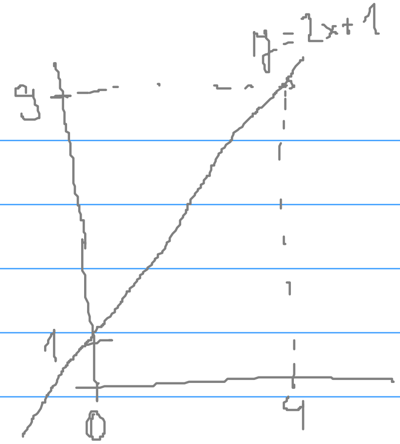


$$\int_a^b f(\varphi(x))\cdot\varphi'(x)\,dx = \int_{\varphi(a)}^{\varphi(b)} f(y)\,dy$$

$y = \varphi(x)$



$$\int_0^4 \sqrt{2x+1}\,dx = \qquad x\ [0,4] \qquad = \frac{1}{2}\int_0^4 \sqrt{2x+1}\;2\,dx \qquad y\ [1,9] \qquad = \frac{1}{2}\int_1^9 \sqrt{y}\,dy$$

$y = 2x+1$
$dy = 2\,dx$

15. $$\int_1^2 \frac{1}{x\cdot\sqrt{x^2+1}}\,dx = \int_1^2 \frac{2x}{2x^2\sqrt{x^2+1}}\,dx = \int_2^5 \frac{1}{2(y-1)\sqrt{y}}\,dy \overset{*}{=}$$

Subst.: $y = x^2+1$ $\qquad x^2 = y-1$ $\qquad x=1 \to y=2$
$dy = 2x\,dx$ $\qquad\qquad x=2 \to y=5$

Druhá subst.

$y = t^2$ $\qquad y=2 \Rightarrow t=\sqrt{2}$ $\qquad t=\sqrt{y}$
$dy = 2t\,dt$ $\qquad y=5 \Rightarrow t=\sqrt{5}$

$$\overset{*}{=} \int_{\sqrt{2}}^{\sqrt{5}} \frac{1}{\cancel{2}(t^2-1)\cancel{t}}\,\cancel{2t}\,dt = \int_{\sqrt{2}}^{\sqrt{5}} \frac{1}{t^2-1}\,dt \overset{*}{=}$$

$$\frac{1}{t^2-1} = \frac{A}{t-1} + \frac{B}{t+1} \qquad 1 = A(t+1) + B(t-1)$$

$$t=-1 \qquad 1 = -2B \qquad B = -\frac{1}{2}$$

$$t=1 \qquad 1 = 2A \qquad A = +\frac{1}{2}$$

$$\overset{*}{=} \int_{\sqrt{2}}^{\sqrt{5}} \frac{1/2}{t-1}\,dt - \int_{\sqrt{2}}^{\sqrt{5}} \frac{\frac{1}{2}}{t+1}\,dt = \frac{1}{2}\Big[\ln|t-1|\Big]_{\sqrt{2}}^{\sqrt{5}} - \frac{1}{2}\Big[\ln|t+1|\Big]_{\sqrt{2}}^{\sqrt{5}} = \frac{1}{2}\Big(\ln(\sqrt{5}-1) - \ln(\sqrt{2}-1) - \ln(\sqrt{5}+1) + \ln(\sqrt{2}+1)\Big)$$

$$= \frac{1}{2}\ln\frac{(\sqrt{5}-1)(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{5}+1)}$$

1. $f(x) = e^{-x^2}\cdot\left(x-\frac{1}{2}\right)$ Int. monotónnosti a L. E.

$D_f = R$

$$f'(x) = e^{-x^2}\cdot(-2x)\cdot\left(x-\frac{1}{2}\right) + e^{-x^2}\cdot 1 = e^{-x^2}\cdot\left(-2x^2+x+1\right)$$

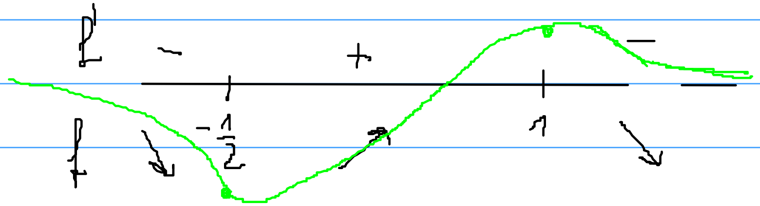
Stac. body $f'(x) = 0 \iff -2x^2+x+1=0$

$2x^2-x-1=0$

$D = 1+8 = 9$ ✓

$x_1 = \frac{1+3}{2\cdot 2} = 1$ ✓

$x_2 = \frac{1-3}{2\cdot 2} = -\frac{1}{2}$ ✓



$f'(2) = e^{-4}(-8+2+1) < 0$

$f'(0) = e^{0}(1) > 0$

$f'(-1) = e^{-1}(-2-1+1) < 0$

Funkcia f je rastúca na $\left\langle -\frac{1}{2}, 1 \right\rangle$

klesajúca na $\left(-\infty, -\frac{1}{2}\right\rangle$ a na $\left\langle 1, \infty\right)$

$x_2 = -\frac{1}{2}$ je bod OLMIN

$x_1 = 1$ —||— OLMAX

$f\left(-\frac{1}{2}\right) = e^{-\left(-\frac{1}{2}\right)^2}\cdot\left(-\frac{1}{2}-\frac{1}{2}\right) = -e^{-\frac{1}{4}} = -\frac{1}{\sqrt[4]{e}}$

$f(1) = e^{-1}\left(1-\frac{1}{2}\right) = \frac{1}{2}\,\frac{1}{e}$

$$\frac{1}{x+1} = (x+1)^{-1}$$

2.

$$\lim_{x\to 0} \frac{x\cdot \ln(x+1)}{1-\cos 2x} \overset{L'H}{=} \lim_{x\to 0} \frac{1\ln(x+1) + x\cdot\frac{1}{x+1}}{+\sin 2x\cdot 2} \overset{L'H}{=} \lim_{x\to 0} \frac{\frac{1}{x+1} + \frac{1}{x+1} + x\frac{-1}{(x+1)^2}}{2\cdot\cos 2x\cdot 2} = \frac{1+1}{4} = \frac{1}{2}$$

$$„\frac{0}{0}" \qquad „\frac{0}{0}"$$

Iný postup

$$\boxed{\sin^2 x = \frac{1-\cos 2x}{2}}$$

$$\lim_{x\to 0} \frac{x\cdot\ln(x+1)}{1-\cos 2x} = \lim_{x\to 0} \frac{x^2\cdot \ln(x+1)}{2\cdot\sin^2 x\cdot x} = \lim_{x\to 0} \underbrace{\frac{x^2}{\sin^2 x}}_{\downarrow\, 1}\cdot\frac{\ln(x+1)}{2x} \overset{L'H}{=} \lim_{x\to 0} \frac{\frac{1}{x+1}}{2} = \frac{1}{2}$$

3. $$\sum_{n=1}^{\infty} \frac{(2n)!}{[(n+1)!]^2 \cdot 3^{2n}}$$

D'Alembert
$$\lim_{n\to\infty} \frac{\dfrac{(2n+2)!}{[(n+2)!]^2 \cdot 3^{2n+2}}}{\dfrac{(2n)!}{[(n+1)!]^2 \cdot 3^{2n}}} = \lim_{n\to\infty} \frac{\cancel{(2n)!}\,(2n+1)(2n+2)}{\cancel{(2n)!}} \cdot \frac{\cancel{3^{2n}}}{\cancel{3^{2n}} \cdot 3^2} \cdot \frac{\cancel{[(n+1)!]^2}}{\cancel{[(n+1)!]^2} \cdot (n+2)^2} =$$

$$= \lim_{n\to\infty} \frac{(2n+1)(2n+2)}{9\,(n+2)^2} = \frac{4}{9} < 1 \implies \text{rad konverguje}$$

4.

$$\int \frac{2x-1}{x^2+4x+5}\,dx =$$

1. Deliť netreba

2. Rozložiť $D = 16-4\cdot1\cdot5 = -4 < 0$ sa nedá

$$= \int \frac{2x+4-5}{x^2+4x+5}\,dx = \int \frac{2x+4}{x^2+4x+5}\,dx - \int \frac{5}{x^2+4x+5}\,dx = \ln|x^2+4x+5| - 5\int \frac{1}{(x^2+4x+4)+1}\,dx =$$

$$\int \frac{\varphi'(x)}{\varphi(x)}\,dx$$

$$x^2+4x+4 = (x+2)^2$$

$$= \ln|x^2+4x+5| - 5\int \frac{1}{(x+2)^2+1^2}\,dx = \ln|x^2+4x+5| - 5\cdot \operatorname{arctg}(x+2) + C$$

5. $$\int_1^e x^2 \cdot \ln^2 x \, dx = \left[\frac{x^3}{3} \cdot \ln^2 x\right]_1^e - \int_1^e \frac{x^3}{3} \cdot 2 \ln x \cdot \frac{1}{x} \, dx =$$

Per partes: $f' = x^2$ $\qquad f = \frac{x^3}{3}$

$g = \ln^2 x \qquad g' = 2 \cdot \ln x \cdot \frac{1}{x}$

$$= \left(\frac{e^3}{3} \cdot 1^2 - \frac{1}{3} \cdot 0^2\right) - \frac{2}{3} \int_1^e x^2 \cdot \ln x \, dx = \frac{e^3}{3} - \frac{2}{3}\left(\left[\frac{x^3}{3} \cdot \ln x\right]_1^e - \int_1^e \frac{x^3}{3} \cdot \frac{1}{x} \, dx\right) =$$

$f' = x^2 \qquad f = \frac{x^3}{3}$

$g = \ln x \qquad g' = \frac{1}{x}$

$$= \frac{e^3}{3} - \frac{2}{3}\left(\frac{e^3}{3} \cdot 1 - 0 - \int_1^e \frac{x^2}{3} \, dx\right) = \frac{e^3}{3} - \frac{2e^3}{9} + \frac{2}{9}\left[\frac{x^3}{3}\right]_1^e = \underline{\underline{\frac{e^3}{3} - \frac{2}{9} e^3 + \frac{2}{27}\left(e^3 - 1\right)}} =$$

$$= \frac{9e^3 - 6e^3 + 2e^3 - 2}{27} = \frac{5}{27} e^3 - \frac{2}{27}$$