$$\frac{\cancel{x^3}}{3}\cdot\frac{1}{\cancel{x}}=\frac{x^2}{3}$$

$$\int x^2\cdot \ln x\, dx = \quad = \frac{x^3}{3}\cdot\ln x-\int\frac{x^3}{3}\cdot\frac{1}{x}\,dx=\frac{x^3}{3}\ln x-\int\frac{x^2}{3}\,dx=\frac{x^3}{3}\ln x-\frac{x^3}{3\cdot 3}+c$$

$f'=x^2$ $\qquad$ $f=\frac{x^3}{3}$

$g=\ln x$ $\qquad$ $g'=\frac{1}{x}$

$$\int\frac{x}{x^2+1}\,dx=\frac{1}{2}\int\frac{2x}{x^2+1}\,dx=\frac{1}{2}\int\frac{1}{y}\,dy=\frac{1}{2}\ln|y|+c=\frac{1}{2}\ln(x^2+1)+c$$

Subst. $y=x^2+1$

$dy=2x\,dx$

$$\int \cos 3x\cdot e^x\,dx=e^x\cdot\cos 3x+3\int e^x\sin 3x\,dx=e^x\cos 3x+3\left(e^x\sin 3x-3\int e^x\cos 3x\,dx\right)$$

$f'=e^x$ $\qquad$ $f=e^x$ $\qquad$ $f'=e^x$ $\qquad$ $f=e^x$

$g=\cos 3x$ $\qquad$ $g'=-3\sin 3x$ $\qquad$ $g=\sin 3x$ $\qquad$ $g'=3\cos 3x$

Označme:

$$I=\int\cos 3x\, e^x\,dx$$

$$I=e^x\cos 3x+3e^x\sin 3x-9I$$

$$10I=e^x(\cos 3x+3\sin 3x)$$

$$I=\frac{1}{10}e^x(\cos 3x+3\sin 3x)=\int\cos 3x\cdot e^x\,dx$$

$$\int \frac{1}{x^2+1}\,dx = \operatorname{arctg} x$$

$$\int \frac{\cos x}{1+\sin^2 x}\,dx = \int \frac{1}{1+y^2}\,dy = \operatorname{arctg} y = \operatorname{arctg}(\sin x) + C$$

Subst.

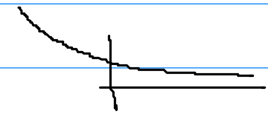
$$\cancel{y = 1+\sin^2 x}$$
$$\cancel{dy = 2\sin x \cos x\,dx}$$

$$y = \sin x$$
$$dy = \cos x\,dx$$

Príklad 1 $f(x) = e^{-x}(x^2+2x-1)$ Nájdite intervaly konvexnosti a inflexné body.

$D_f = R$ $\qquad (e^{-x})' = e^{-x}(-1)$

$$f'(x) = -e^{-x}(x^2+2x-1) + e^{-x}(2x+2) = e^{-x}(-x^2+3)$$



$$f''(x) = -e^{-x}(-x^2+3) + e^{-x}(-2x) = e^{-x}(x^2-2x-3)$$

Kedy $f''(x) = 0$? $\qquad x^2-2x-3=0 \qquad x_{1,2} = \frac{2 \pm \sqrt{4-4\cdot 1\cdot(-3)}}{2} = 1 \pm 2 = \begin{cases} 3 \\ -1 \end{cases}$

	$(-\infty,-1)$	-1	$(-1,3)$	3	$(3,\infty)$
f''	$+$		$-$		$+$
f	$\cup$		$\cap$		$\cup$

$f''(4) = e^{-4}(16-8-3) = 5e^{-4} > 0$

$f''(0) = 1\cdot(-3) < 0$

$f''(-2) = e^{2}(4+4-3) = 5e^{2} > 0$

Konvexná na $(-\infty,-1\rangle$ a na $\langle 3,\infty)$

konkávna na $\langle -1,3\rangle$.

Inflexné body sú $x_1=-1$ a $x_2=3$.

Pr 2.

"$\infty - \infty$"

$$\lim_{x\to\infty} \ln x - \sqrt{x} = \lim_{x\to\infty} \frac{(\ln x - \sqrt{x})\cdot(\ln x+\sqrt{x})}{(\ln x+\sqrt{x})} = \lim_{x\to\infty}\frac{\ln^2 x - x}{\ln x+\sqrt{x}} \overset{L'H}{=}$$

$$= \lim_{x\to\infty} \frac{2\cdot\ln x\cdot\frac{1}{x} - 1}{\frac{1}{x}+\frac{1}{2\sqrt{x}}}\cdot\frac{x}{x} = \lim_{x\to\infty} \frac{2\ln x - x}{1+\frac{1}{2}\sqrt{x}} \overset{L'H}{=} \lim_{x\to\infty} \frac{2\cdot\frac{1}{x} - 1}{\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{x}}} = \lim_{x\to\infty} \frac{8\cdot\frac{1}{x} - 4}{\frac{1}{\sqrt{x}}} =$$

$$= \lim_{x\to\infty} 8\cdot\underbrace{\frac{1}{\sqrt{x}}}_{0} - 4\underbrace{\sqrt{x}}_{\infty} = -\infty$$

Pr 3. a) Súčet radu

$$\sum_{n=1}^{\infty} \frac{2\cdot 3^n - 1}{2^{3n}} = \sum_{n=1}^{\infty} 2\cdot \frac{3^n}{8^n} - \sum_{n=1}^{\infty} \frac{1}{8^n} = \frac{\frac{6}{8}}{1-\frac{3}{8}} - \frac{\frac{1}{8}}{1-\frac{1}{8}} = \frac{6}{5} - \frac{1}{7}$$

$a_1 = \frac{6}{8}$ $\qquad a_2 = \frac{1}{8}$

$q_1 = \frac{3}{8}$ $\qquad q_2 = \frac{1}{8}$

b) $$\sum_{n=1}^{\infty} \frac{1}{n^2+2n} = \sum_{n=1}^{\infty} \left(\frac{1/2}{n} - \frac{1/2}{n+2} \right) = \frac{3}{4}$$

$$a_n = \frac{1}{n^2+2n} = \frac{A}{n} + \frac{B}{n+2} \qquad 1 = A\cdot(n+2) + B\cdot n$$

$n^2+2n = n(n+2)$

$n=0 \qquad 1 = A\cdot 2 \qquad A = \frac{1}{2}$

$n=-2 \qquad 1 = B(-2) \qquad B = -\frac{1}{2}$

$$S_N = \left(\frac{1/2}{1} - \cancel{\frac{1/2}{3}}\right) + \left(\frac{1/2}{2} - \cancel{\frac{1/2}{4}}\right) + \left(\cancel{\frac{1/2}{3}} - \cancel{\frac{1/2}{5}}\right) + \left(\cancel{\frac{1/2}{4}} - \cancel{\frac{1/2}{6}}\right) + \left(\cancel{\frac{1/2}{5}} - \cancel{\frac{1/2}{7}}\right) + \dots + \left(\cancel{\frac{1/2}{N-1}} - \frac{1/2}{N+1}\right) + \left(\cancel{\frac{1/2}{N}} - \frac{1/2}{N+2}\right)$$

$$\boxed{S_N = \frac{1}{2} + \frac{1}{4} - \frac{1/2}{N+1} - \frac{1/2}{N+2}}$$

N-tý čiastočný súčet

$$S = \lim_{N\to\infty} S_N = \lim_{N\to\infty} \frac{3}{4} - \frac{1/2}{N+1} - \frac{1/2}{N+2} = \frac{3}{4}$$

Pr 4 $\int \operatorname{arctg}\sqrt{x}\,dx = 2\int \operatorname{arctg}(y)\cdot y\,dy = 2\cdot\left(\frac{y^2}{2}\cdot\operatorname{arctg} y - \int \frac{y^2}{2}\cdot\frac{1}{1+y^2}\,dy\right) =$

Subst. $\sqrt{x} = y$

$x = y^2$

$dx = 2y\,dy$

Per partes

$g = \operatorname{arctg} y$ $\quad g' = \frac{1}{1+y^2}$

$f' = y$ $\quad f = \frac{y^2}{2}$

$$= y^2\cdot\operatorname{arctg} y - \int \frac{y^2}{y^2+1}\,dy = y^2\operatorname{arctg} y - \int 1 - \frac{1}{y^2+1}\,dy = y^2\operatorname{arctg} y - y + \operatorname{arctg} y + c =$$

Dělit $y^2 : (y^2+1) = 1 - \frac{1}{y^2+1}$

$-(y^2+1)$

-1

$$= x\operatorname{arctg}\sqrt{x} - \sqrt{x} + \operatorname{arctg}\sqrt{x} + c$$

Př5. $\displaystyle\int_{-2}^{0} \frac{x^2+2}{x^2+4x+8}\,dx = \int_{-2}^{0} 1 - \frac{4x+6}{x^2+4x+8}\,dx =$

Má menovateľ koreň?

$x^2+4x+8=0$

$D = 16-4\cdot 1\cdot 8 < 0$ NEMÁ

Deliť: $x^2+2 : (x^2+4x+8) = 1 - \dfrac{4x+6}{x^2+4x+8}$

$-(x^2+4x+8)$

$-4x-6$

$$= [x]_{-2}^{0} - 2\int_{-2}^{0} \frac{2x+3+1-1}{x^2+4x+8}\,dx = (0-(-2)) - 2\int_{-2}^{0} \frac{2x+4}{x^2+4x+8}\,dx + 2\int_{-2}^{0} \frac{1}{x^2+4x+8}\,dx =$$

$$= 2 - 2\left[\ln|\overset{2x+4}{x^2+4x+8}|\right]_{-2}^{0} + 2\int_{-2}^{0} \frac{1}{x^2+4x+4+4}\,dx = 2 - 2(\ln 8 - \ln 4) + 2\int_{-2}^{0} \frac{1}{(x+2)^2+2^2}\,dx =$$

$x^2+4x+4 = (x+2)^2$

$$= 2 - 2\ln\frac{8}{4} + 2\left[\frac{1}{2}\operatorname{arctg}\frac{x+2}{2}\right]_{-2}^{0} = 2 - 2\ln 2 + \underbrace{\operatorname{arctg} 1}_{\frac{\pi}{4}} - \underbrace{\operatorname{arctg} 0}_{0} = 2 - 2\ln 2 + \frac{\pi}{4}$$

$$\int \frac{\varphi'(x)}{\varphi(x)}\,dx = \ln|\varphi(x)|$$

$\varphi'(x) = 2x+4$

$\varphi(x) = x^2+4x+8$