1 Príklad $\quad D_f = R - \{-2,2\}$

$$f'(x) = \frac{2(4-x^2) - (2x+5)(-2x)}{(4-x^2)^2} = \frac{8-2x^2+4x^2+10x}{(4-x^2)^2} = \frac{2x^2+10x+8}{(4-x^2)^2}$$

$$f'(x)=0 \iff x^2+5x+4=0 \iff x=-1 \quad \vee \quad x=-4$$

f'	$+$		$-$		$-$		$+$		$+$
f	$\nearrow$	-4	$\searrow$	-2	$\searrow$	-1	$\nearrow$	2	$\nearrow$

f je klesajúca na $\langle -4,-2)$ a $(-2,-1\rangle$

rastúca na $(-\infty,-4\rangle$, $\langle -1,2)$ a $(2,\infty)$

$x_1=-1$ je bod OLMIN

$x_2=-4$ je bod OLMAX

b, $\displaystyle\lim_{x\to 2+} \frac{2x+5}{4-x^2} = -\infty$

Pr 2. a, D'Alembertovo kritérium

$$\lim_{x\to\infty} \frac{\frac{(n+3)!\cdot(n+1)!\cdot 2^{n+1}}{(2n+2)!}}{\frac{(n+2)!\cdot n!\cdot 2^n}{(2n)!}} = \frac{\cancel{(n+2)!}(n+3)}{\cancel{(n+2)!}} \cdot \frac{\cancel{n!}(n+1)}{\cancel{n!}} \cdot \frac{\cancel{2^n}\cdot 2}{\cancel{2^n}} \cdot \frac{\cancel{(2n)!}}{\cancel{(2n)!}(2n+1)(2n+2)} =$$

$$= \lim_{x\to\infty} \frac{(n+3)(n+1)\cdot 2}{(2n+1)(2n+2)} = \frac{2}{4} = \frac{1}{2} < 1 \implies \text{rad konverguje}$$

b, $\displaystyle\sum_{n=1}^{\infty} \frac{3\cdot 5^{n+1}}{4^{2n}} = \frac{3\cdot 5^2}{16}\cdot\frac{1}{1-\frac{5}{16}} = \frac{3\cdot 25}{\cancel{16}}\cdot\frac{\cancel{16}}{11} = \frac{75}{11}$

Pr 3. $\displaystyle\int x \operatorname{arctg} x \, dx = \frac{x^2}{2}\cdot\operatorname{arctg} x - \int \frac{x^2}{2}\cdot\frac{1}{1+x^2}\,dx =$

$f' = x \qquad f = \frac{x^2}{2}$

$g = \operatorname{arctg} x \qquad g' = \frac{1}{1+x^2}$

$$= \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}\int \frac{x^2+1-1}{1+x^2}\,dx = \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}\left(\int 1\,dx - \int \frac{1}{1+x^2}\,dx\right) =$$

$$= \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}x + \frac{1}{2}\operatorname{arctg} x + c$$

Pr 4. $\int_0^8 \frac{1}{1+\sqrt[3]{x}}\,dx = \int_0^2 \frac{1}{1+t}\,3t^2\,dt = 3\int_0^2 \frac{t^2}{1+t}\,dt = 3\int_0^2 t-1+\frac{1}{t+1}\,dt$

Subst. $x = t^3$

$dx = 3t^2\,dt$

$t^2 : (t+1) = t-1+\frac{1}{t+1}$

$= 3\left[\frac{t^2}{2}\right]_0^2 - 3\left[t\right]_0^2 + 3\left[\ln|t+1|\right]_0^2 = 3 \cdot 2 - 3 \cdot 2 + 3\ln 3 = 3 \cdot \ln 3$