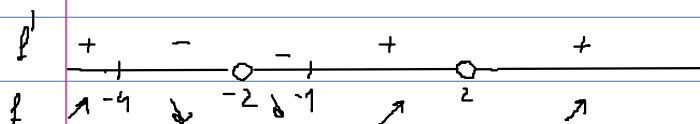


1 Příklad $D_f = \mathbb{R} - \{-2, 2\}$

$$f'(x) = \frac{2(4-x^2) - (2x+5)(-2x)}{(4-x^2)^2} = \frac{8 - 2x^2 + 4x^2 + 10x}{(4-x^2)^2} = \frac{2x^2 + 10x + 8}{(4-x^2)^2}$$

$$f'(x) = 0 \Leftrightarrow x^2 + 5x + 4 = 0 \Leftrightarrow x = -1 \vee x = -4$$



f je klesající na $(-4, -2)$ a $(-2, -1)$

rostoucí na $(-\infty, -4)$, $(-1, 2)$ a $(2, \infty)$

$x_1 = -1$ je bod OLMIN

$x_2 = -4$ je bod OLMAX

$$b.) \lim_{x \rightarrow 2^+} \frac{2x+5}{4-x^2} = -\infty$$

Pr 2. a.) D'Alembertovo kritérium

$$\lim_{n \rightarrow \infty} \frac{(n+3)! \cdot (n+1)! \cdot 2^{n+1}}{(2n+2)! \cdot n! \cdot 2^n} = \frac{\cancel{(n+2)!} (n+3) \cdot \cancel{n!} (n+1) \cdot 2 \cdot 2 \cdot \cancel{(2n)!}}{\cancel{(2n)!} \cdot \cancel{n!} \cdot 2^n} =$$

$$= \lim_{n \rightarrow \infty} \frac{(n+3)(n+1) \cdot 2}{(2n+1)(2n+2)} = \frac{2}{4} = \frac{1}{2} < 1 \Rightarrow \text{rad konvergence}$$

$$b.) \sum_{m=1}^{\infty} \frac{3 \cdot 5^{m+1}}{4^{2m}} = \frac{3 \cdot 5^2}{16} \cdot \frac{1}{1 - \frac{5}{16}} = \frac{3 \cdot 25}{16} \cdot \frac{16}{11} = \frac{75}{11}$$

Pr 3.

$$\int x \arctg x \, dx = \frac{x^2}{2} \cdot \arctg x - \int \frac{x^2}{2} \cdot \frac{1}{1+x^2} dx =$$
$$f' = x \quad f = \frac{x^2}{2}$$
$$g = \arctg x \quad g' = \frac{1}{1+x^2}$$

$$= \frac{x^2}{2} \arctg x - \frac{1}{2} \int \frac{x^2 + 1 - 1}{1+x^2} dx = \frac{x^2}{2} \arctg x - \frac{1}{2} \left(\int 1 dx - \int \frac{1}{1+x^2} dx \right) =$$
$$= \frac{x^2}{2} \arctg x - \frac{1}{2} x + \frac{1}{2} \arctg x + c$$

$$\text{Pr 4. } \int_0^8 \frac{1}{1+\sqrt[3]{x}} dx = \int_0^2 \frac{1}{1+t} 3t^2 dt = 3 \int_0^2 \frac{t^2}{1+t} dt = 3 \int_0^2 t-1 + \frac{1}{t+1} dt$$

$$\text{Subst. } x = t^3 \\ dx = 3t^2 dt$$

$$t^2 : (t+1) = t-1 + \frac{1}{t+1}$$

$$= 3 \left[\frac{t^2}{2} \right]_0^2 - 3 [t]_0^2 + 3 \left[\ln|t+1| \right]_0^2 = 3 \cdot 2 - 3 \cdot 2 + 3 \ln 3 = 3 \cdot \ln 3$$