Riešenie 1. a, Definičný obor je $D_f = R - \{-2, 2\}$

Vypočítame $f'(x) = \dfrac{2(4-x^2)+(2x+5)2x}{(4-x^2)^2} = \dfrac{2x^2+10x+8}{(4-x^2)^2}$

$f(x) = \dfrac{2x+5}{4-x^2} = 0$ práve keď $x = -\dfrac{5}{2}$. Bod dotyku je $\left[-\dfrac{5}{2}, 0\right]$.

$$f'\left(-\frac{5}{2}\right) = \frac{2\cdot\frac{25}{4} - 10\cdot\frac{5}{2} + 8}{\left(4-\frac{25}{4}\right)^2} = \frac{-\frac{25}{2}+\frac{16}{2}}{\left(-\frac{9}{4}\right)^2} = -\frac{\frac{9}{2}}{\frac{81}{16}} = -\frac{8}{9}$$

Rovnica dotyčnice je $y - 0 = -\dfrac{8}{9}\left(x + \dfrac{5}{2}\right)$.

b,

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{2+\frac{5}{x}}{x\left(-1+\frac{4}{x^2}\right)} = 0$$

$$\lim_{x \to 2+} f(x) = -\infty$$

c) Stacionárne body sú korene rovnice $2x^2+10x+8=0$

To sú

$$x_{1,2}=\frac{-10\pm\sqrt{100-64}}{4}=\left\langle\begin{matrix}-4\\ -1\end{matrix}\right.$$

Vypočítame znamienko derivácie na každom z intervalov na obrázku

		-4		-2		-1		2	
f'	$+$		$-$		$-$		$+$		$+$
f	$\nearrow$		$\searrow$		$\searrow$		$\nearrow$		$\nearrow$

$f'(10)>0$ $\quad f'(-10)>0$ $\quad f'(0)>0$

$f'(-3)<0$ $\quad f'\left(-\frac{3}{2}\right)<0$

Funkcia f je rastúca na $(-\infty,-4\rangle, \langle-1,2), (2,\infty)$

klesajúca na $\langle-4,-2)$ a $(-2,-1\rangle$.

bod -4 je bodom OLMAX

-1 —||— OLMIN.

Riešenie 2.

a) Použijeme D'Alembertovo kritérium

$$\lim_{n\to\infty} \frac{\dfrac{(2n+2)!\, 2^{2n+3}\,(n+1)!}{(3n+1)!}}{\dfrac{(2n)!\, 2^{2n+1}\, n!}{(3n-2)!}} = \lim_{n\to\infty} \frac{\cancel{(2n)!}(2n+1)(2n+2)\cancel{2^{2n+1}}\cdot 4\cdot \cancel{n!}(n+1)\cancel{(3n-2)!}}{\cancel{(2n)!}\,\cancel{2^{2n+1}}\cdot\cancel{n!}\,\cancel{(3n-2)!}\,(3n-1)3n(3n+1)} =$$

$$= \lim_{n\to\infty} \frac{\cancel{n}\left(2+\frac{1}{n}\right)\cdot \cancel{n}\left(2+\frac{2}{n}\right)\cdot 4\cdot \cancel{n}\left(1+\frac{1}{n}\right)}{\cancel{n}\left(3-\frac{1}{n}\right)3\cancel{n}\cdot \cancel{n}\left(3+\frac{1}{n}\right)} = \frac{16}{27} < 1 \Rightarrow$$

$\Rightarrow$ rad konverguje

b) Použijeme porovnávacie kritérium

Rad porovnáme s radom $\sum \frac{1}{n}$

$$\lim_{n\to\infty} \frac{\dfrac{\sqrt{n^2+1}}{2n^2-n+2}}{\dfrac{1}{n}} = \lim_{n\to\infty} \frac{\cancel{n\cdot\sqrt{n^2}}\cdot\sqrt{1+\frac{1}{n^2}}}{\cancel{n^2}\left(2-\frac{1}{n}+\frac{2}{n^2}\right)} = \frac{1}{2} > 0$$ a konečná, preto

oba rady majú rovnaký charakter konvergencie.

Rad $\sum\limits_{n=1}^{\infty} \frac{1}{n}$ diverguje, preto aj zadaný rad diverguje.

Riešenie 4

a.) $\displaystyle\int_1^8 \frac{1+\sqrt[3]{x}}{1+\sqrt[3]{x^2}}\,dx \overset{*}{=}$ Použijeme substitúciu $x = t^3$, $dx = 3t^2\,dt$ $8 = 2^3$, $1 = 1^3$

$$\overset{*}{=} \int_1^2 \frac{1+t}{1+t^2}\,3t^2\,dt = 3\int_1^2 \frac{t^3+t^2}{t^2+1}\,dt \overset{**}{=}$$

Delením dostaneme

$$t^3+t^2 : (t^2+1) = t+1 - \frac{t+1}{t^2+1}$$

$$\overset{*}{=} 3\int_1^2 t+1-\frac{t+1}{t^2+1}\,dt = 3\left[\frac{t^2}{2}+t\right]_1^2 - \frac{3}{2}\int_1^2 \frac{2t}{t^2+1}\,dt - 3\int_1^2 \frac{1}{t^2+1}\,dt = 3\left(4-\frac{3}{2}\right) - \frac{3}{2}\left[\ln(t^2+1)\right]_1^2 -$$

$$-3\left[\operatorname{arctg} t\right]_1^2 = \frac{15}{2} - \frac{3}{2}\ln\frac{5}{2} - 3\operatorname{arctg} 2 + 3\operatorname{arctg} 1$$

b.) Ak je dolná hranica $0 < a < 8$, tak

$$\int_a^8 \frac{1+\sqrt[3]{x}}{1+\sqrt[3]{x^2}}\,dx = 3\int_{\sqrt[3]{a}}^2 t+1-\frac{1}{2}\frac{2t}{t^2+1}-\frac{1}{t^2+1}\,dt = 3\left(4-\frac{a^{\frac{2}{3}}}{2}-a^{\frac{1}{3}}\right) - \frac{3}{2}\ln\frac{5}{a^{\frac{2}{3}}+1} - 3\operatorname{arctg} 2 +$$

$$+3\operatorname{arctg} a^{\frac{1}{3}}$$

Riešenie 3 Počítame neurčitý integrál

$$F(x) = \int x^3 e^{2x^2} dx =$$

Použijeme substitúciu $y = x^2$

$dy = 2x dx$

$$= \int y e^{2y} \frac{1}{2} dy =$$

Pokračujeme metódou PerPartes

$f' = e^{2y}$ $g = y$

$f = \frac{1}{2} e^{2y}$ $g' = 1$

$$= \frac{1}{4} e^{2y} \cdot y - \frac{1}{4} \int e^{2y} dy = \frac{1}{4} e^{2y} \cdot y - \frac{1}{8} e^{2y} + c = \frac{1}{4} e^{2x^2} \cdot x^2 - \frac{1}{8} e^{2x^2} + c$$

$F(0) = -\frac{1}{8} + c$ Aby $F(0) = 0$ musí byť $c = \frac{1}{8}$

$$F(x) = \frac{1}{4} e^{2x^2} \cdot x^2 - \frac{1}{8} e^{2x^2} + \frac{1}{8}$$