Pr 1. $f(x) = e^{2x}(x^2-2x-1)$ $\qquad D_f = \mathbb{R}$

a) $f'(x) = 2e^{2x}(x^2-2x-1) + e^{2x}(2x-2) = e^{2x}(2x^2-2x-4) = 2e^{2x}(x^2-x-2)$

St. body: $f'(x) = 0 \iff x^2-x-2=0$

Korene $x_1=-1 \qquad x_2 = 2$

f': $+$ on $(-\infty,-1)$ ↗, $-$ on $(-1, 2)$ ↘, $+$ on $(2,\infty)$ ↗

Rastúca na $(-\infty,-1\rangle$ a na $\langle 2,\infty)$

Klesajúca na $\langle -1, 2\rangle$.

$x_1=-1$ je bod OLMAX $\qquad x_2=2$ je bod OLMIN.

b) $$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} e^{2x}(x^2-2x-1) = \lim_{x\to-\infty}\frac{x^2-2x-1}{e^{-2x}} = \lim_{x\to-\infty}\frac{2x-2}{-2e^{-2x}} = \lim_{x\to-\infty}\frac{2}{4\cdot e^{2x}} = 0$$

Pr 2. a)

$$\sum_{n=2}^{\infty}\frac{4\cdot 3^{2n}+(-1)^n}{2^{4n}} = \sum_{n=2}^{\infty} 4\,\frac{9^n}{16^n} + \sum_{n=2}^{\infty}\left(\frac{-1}{16}\right)^n$$

Pretože $q_1 = \frac{9}{16} \qquad q_2 = -\frac{1}{16}$

$\left|\frac{9}{16}\right| < 1 \qquad \left|-\frac{1}{16}\right| < 1$

obe geom. rady konvergujú.

$a_{01} = 4\cdot\left(\frac{9}{16}\right)^2 \qquad a_{02} = \left(-\frac{1}{16}\right)^2$

Preto $$\sum_{n=2}^{\infty}\frac{4\cdot 3^{2n}+(-1)^n}{2^{4n}} = 4\left(\frac{9}{16}\right)^2\cdot\frac{1}{1-\frac{9}{16}} + \left(-\frac{1}{16}\right)^2\cdot\frac{1}{1+\frac{1}{16}} = \frac{4\cdot 9^2}{16^2}\cdot\frac{16}{7} + \frac{1}{16^2}\cdot\frac{16}{17} =$$

$$= \frac{81}{28} + \frac{1}{16\cdot 17}$$

b) Použijeme Cauchyho kritérium

$$\lim_{n\to\infty}\sqrt[n]{a_n} = \lim_{n\to\infty}\frac{n^2+3n+1}{(2n+1)^2} = \lim_{n\to\infty}\frac{n^2+3n+1}{4n^2+4n+1} = \frac{1}{4} < 1$$ preto rad konverguje.

Pr 3
$$\int x \cdot \ln(x^2+2)\,dx =$$

Metóda per partes: $f' = x$ $\quad f = \frac{x^2}{2}$

$g = \ln(x^2+2)$ $\quad g' = \frac{1}{x^2+2}\cdot 2x$

$$= \frac{x^2}{2}\ln(x^2+2) - \int \frac{x^2}{2}\cdot\frac{2x}{x^2+2}\,dx = \frac{x^2}{2}\ln(x^2+2) - \int \frac{x^3}{x^2+2}\,dx \overset{*}{=}$$

$$x^3 : x^2+2 = x - \frac{2x}{x^2+2}$$
$$-(x^3+2x)$$
$$-2x$$

$$\overset{*}{=} \frac{x^2}{2}\ln(x^2+2) - \int x - \frac{2x}{x^2+2}\,dx =$$

$$= \frac{x^2}{2}\ln(x^2+2) - \frac{x^2}{2} + \ln(x^2+2) + c$$

Pr 4.
$$\int_1^8 \frac{\sqrt[3]{x}+1}{1+\sqrt[3]{x^2}}\,dx =$$

Subst.: $x = t^3$ $\quad x=1 \rightarrow t=1$ $\quad (t = \sqrt[3]{x})$

$dx = 3t^2\,dt$ $\quad x=8 \rightarrow t=2$

$$= \int_1^2 \frac{t+1}{1+t^2}3t^2\,dt = 3\int_1^2 \frac{t^3+t^2}{t^2+1}\,dt = 3\int_1^2 t+1-\frac{t}{t^2+1}-\frac{1}{t^2+1}\,dt =$$

$$t^3+t^2 : (t^2+1) = t+1-\frac{t+1}{t^2+1}$$
$$-(t^3+t)$$
$$t^2-t$$
$$-(t^2+1)$$
$$-t-1$$

$$= 3\left[\frac{t^2}{2}+t-\frac{1}{2}\ln(t^2+1)-\operatorname{arctg} t\right]_1^2 = 3\left(2+2-\frac{1}{2}\ln 5-\operatorname{arctg}2-\frac{1}{2}-1+\frac{1}{2}\ln 2+\right.$$

$$\left.+\operatorname{arctg}1\right) = 3\left(\frac{5}{2}+\frac{1}{2}\ln\frac{2}{5}-\operatorname{arctg}2+\frac{\pi}{4}\right)$$