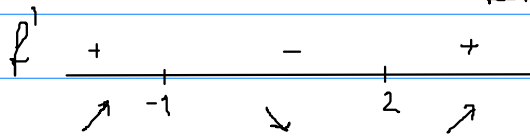


Pr1. $f(x) = e^{2x}(x^2 - 2x - 1)$ $D_f = \mathbb{R}$

a) $f'(x) = 2e^{2x}(x^2 - 2x - 1) + e^{2x}(2x - 2) = e^{2x}(2x^2 - 2x - 4) = 2e^{2x}(x^2 - x - 2)$

St body: $f'(x) = 0 \iff x^2 - x - 2 = 0$

korene $x_1 = -1$ $x_2 = 2$



Rostúca na $(-\infty, -1)$ a na $(2, \infty)$

Klesajúca na $(-1, 2)$.

$x_1 = -1$ je bod OLMAX

$x_2 = 2$ je bod OLMIN.

b) $\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} e^{2x}(x^2 - 2x - 1) = \lim_{x \rightarrow -\infty} \frac{x^2 - 2x - 1}{e^{-2x}} = \lim_{x \rightarrow -\infty} \frac{2x - 2}{-2e^{-2x}} = \lim_{x \rightarrow -\infty} \frac{2}{4e^{2x}} = 0$

Pr2.

a)

$$\sum_{m=2}^{\infty} \frac{4 \cdot 3^{2m} + (-1)^m}{2^{4m}} = \sum_{m=2}^{\infty} 4 \frac{9^m}{16^m} + \sum_{m=2}^{\infty} \left(\frac{-1}{16}\right)^m$$

Pretože $a_1 = \frac{9}{16}$ $a_2 = -\frac{1}{16}$

$$\left|\frac{9}{16}\right| < 1 \quad \left|-\frac{1}{16}\right| < 1$$

obidva geom. rady konvergujú.

$$a_{01} = 4 \cdot \left(\frac{9}{16}\right)^2 \quad a_{02} = \left(-\frac{1}{16}\right)^2$$

Preto $\sum_{m=2}^{\infty} \frac{4 \cdot 3^{2m} + (-1)^m}{2^{4m}} = 4 \left(\frac{9}{16}\right)^2 \cdot \frac{1}{1 - \frac{9}{16}} + \left(-\frac{1}{16}\right)^2 \cdot \frac{1}{1 + \frac{1}{16}} = \frac{4 \cdot 9^2}{16^2} \cdot \frac{16}{7} + \frac{1}{16^2} \cdot \frac{16}{17} =$

$$= \frac{81}{28} + \frac{1}{16 \cdot 17}$$

b) Použijeme Cauchyho kritérium

$$\lim_{m \rightarrow \infty} \sqrt[m]{a_m} = \lim_{m \rightarrow \infty} \frac{m^2 + 3m + 1}{(2m+1)^2} = \lim_{m \rightarrow \infty} \frac{m^2 + 3m + 1}{4m^2 + 4m + 1} = \frac{1}{4} < 1 \text{ preto rad konverguje.}$$

$$Pr 3 \quad \int x \cdot \ln(x^2+2) dx =$$

Método $f' = x$ $f = \frac{x^2}{2}$
 por partes $g = \ln(x^2+2)$ $g' = \frac{1}{x^2+2} \cdot 2x$

$$= \frac{x^2}{2} \ln(x^2+2) - \int \frac{x^2}{2} \cdot \frac{2x}{x^2+2} dx = \frac{x^2}{2} \ln(x^2+2) - \int \frac{x^3}{x^2+2} dx = *$$

$$\begin{array}{l} x^3 : x^2+2 = x - \frac{2x}{x^2+2} \\ -(x^2+2) \\ -2x \end{array}$$

$$= \frac{x^2}{2} \ln(x^2+2) - \int x - \frac{2x}{x^2+2} dx =$$

$$= \frac{x^2}{2} \ln(x^2+2) - \frac{x^2}{2} + \ln(x^2+2) + c$$

$$Pr 4. \quad \int_1^8 \frac{\sqrt[3]{x}+1}{1+\sqrt[3]{x^2}} dx =$$

Subst.: $x = t^3$ $x=1 \rightarrow t=1$ $(t = \sqrt[3]{x})$
 $dx = 3t^2 dt$ $x=8 \rightarrow t=2$

$$= \int_1^2 \frac{t+1}{1+t^2} 3t^2 dt = 3 \int_1^2 \frac{t^3+t^2}{t^2+1} dt = 3 \int_1^2 t+1 - \frac{t}{t^2+1} - \frac{1}{t^2+1} dt =$$

$$\begin{array}{l} t^3+t^2 : (t^2+1) = t+1 - \frac{t+1}{t^2+1} \\ -(t^3+t) \\ \frac{t^2-t}{t^2+1} \\ \frac{-t-1}{-t-1} \end{array}$$

$$= 3 \left[\frac{t^2}{2} + t - \frac{1}{2} \ln(t^2+1) - \operatorname{arctg} t \right]_1^2 = 3 \left(2+2 - \frac{1}{2} \ln 5 - \operatorname{arctg} 2 - \frac{1}{2} - 1 + \frac{1}{2} \ln 2 + \right.$$

$$\left. + \operatorname{arctg} 1 \right) = 3 \left(\frac{5}{2} + \frac{1}{2} \ln \frac{2}{5} - \operatorname{arctg} 2 + \frac{\pi}{4} \right)$$