$$\sum_{n=2}^{\infty} \frac{3\cdot(-2)^{2n+1}}{5^{n-1}} = \sum_{n=2}^{\infty} 3 \cdot \frac{(-2)^{2n}\cdot(-2)}{5^{n}\cdot 5^{-1}} = \sum_{n=2}^{\infty} 3\cdot 5\cdot(-2)\left(\frac{4}{5}\right)^{n} = -30\cdot\frac{16}{25}\cdot\frac{1}{1-\frac{4}{5}} = -\frac{6\cdot 16}{\cancel{5}}\cdot\cancel{5} = -96$$

pre $n=2$ $\quad a = 3\cdot 5\cdot(-2)\cdot\left(\frac{4}{5}\right)^2 \quad q = \frac{4}{5}$

Kritéria konvergencie pre rady s nezápornými členmi.

1. Cauchy

Pr1. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2n}{3n+1}\right)^{2n-3}$

$\lim\limits_{n\to\infty}\sqrt[n]{a_n} = l$ $\begin{cases} 0 < l < 1 & \text{rad konverguje} \\ 1 < l & \text{rad diverguje.} \end{cases}$

$$\lim_{n\to\infty}\sqrt[n]{\left(\frac{2n}{3n+1}\right)^{2n-3}} = \lim_{n\to\infty}\left(\underbrace{\frac{2n}{3n+1}}_{\to\frac{2}{3}}\right)^{\underbrace{\frac{2n-3}{n}}_{\to 2}} = \left(\frac{2}{3}\right)^2 = \frac{4}{9} < 1 \Rightarrow \text{rad konverguje}$$

Pr2. $\sum_{n=0}^{\infty}\left(\frac{\operatorname{arctg} n}{2}\right)^{n}$

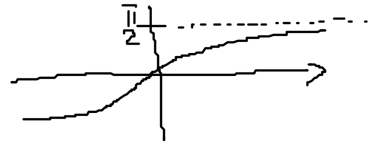
$\lim_{n\to\infty} \operatorname{arctg} n$

∞

0

π 1

$\frac{\pi}{2}$ $||||||$



$$\lim_{n\to\infty}\sqrt[n]{\left(\frac{\operatorname{arctg} n}{2}\right)^{n}} = \lim_{n\to\infty}\left(\frac{\operatorname{arctg} n}{2}\right)^{\cancel{n}\cdot\frac{1}{\cancel{n}}} = \frac{\pi}{4} < 1 \Rightarrow \text{rad konverguje}$$

Pr2b. $\sum_{n=0}^{\infty}\left(\frac{2\operatorname{arctg} n}{3}\right)^{n}$

$$= \lim_{n\to\infty}\left(\frac{2\operatorname{arctg} n}{3}\right)^{\cancel{n}\cdot\frac{1}{\cancel{n}}} = \frac{2}{3}\cdot\frac{\pi}{2} = \frac{\pi}{3} > 1 \Rightarrow \text{rad diverguje}$$

Pr3. $\sum_{n=1}^{\infty}\frac{3^{n}-1}{2^{2n+1}+7}$

$$\lim_{n\to\infty}\sqrt[n]{\frac{3^{n}-1}{2^{2n+1}+7}} < \lim_{n\to\infty}\sqrt[n]{\frac{3^{n}}{2^{2n+1}}} = \lim_{n\to\infty}\frac{3}{4\cdot\sqrt[n]{2}} = \frac{3}{4}\lim_{n\to\infty}\frac{1}{\sqrt[n]{2}} = \frac{3}{4} < 1 \text{ rad konverguje}$$

Pr4. $\sum_{n=1}^{\infty}\left(\frac{n}{n+2}\right)^{n^{2}+n}$

$$\lim_{n\to\infty}\left(\frac{n}{n+2}\right)^{\frac{n^{2}+n}{n}} = \lim_{n\to\infty}\left(\frac{n}{n+2}\right)^{n+1}$$

$$\lim_{x\to\infty}\left(\frac{x}{x+2}\right)^{x+1} =$$

Príprava $\frac{x}{x+2} = e^{\ln\frac{x}{x+2}}$

$$= \lim_{x\to\infty} e^{\left(\ln\frac{x}{x+2}\right)\cdot(x+1)} =$$

limita exponentu $\lim_{x\to\infty}\left(\underset{0}{\ln\frac{x}{x+2}}\right)\cdot\underset{\infty}{(x+1)} = \lim_{x\to\infty}\frac{\ln\frac{x}{x+2}}{\frac{1}{x+1}} \overset{L'H}{=}$ $\frac{0}{0}$

$$= \lim_{x\to\infty}\frac{\frac{1}{\frac{x}{x+2}}\cdot\frac{(x+2)-x\cdot 1}{(x+2)^2}}{-1\cdot\frac{1}{(x+1)^2}\cdot 1} = \lim_{x\to\infty}\frac{\frac{\cancel{x+2}}{x}\cdot\frac{2}{(x+2)^{\cancel{2}\,1}}\,(x+1)^2}{-1} =$$

$$= \lim_{x\to\infty}\frac{2\cdot(x^2+2x+1)}{-1\,(x^2+2x)} = -2$$

$$= e^{-2} = \frac{1}{e^2} < 1 \Rightarrow \text{rad konverguje.}$$

2. D'Alembertovo kritérium

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = l \;\Big\langle$$

$0 < l < 1$ rad konv.

$l > 1$ rad div.

Pr 5. $\sum_{n=1}^{\infty}\frac{n^3}{3^n}$

$$\lim_{n\to\infty}\frac{\frac{(n+1)^3}{3^{n+1}}}{\frac{n^3}{3^n}} = \lim_{n\to\infty}\frac{\cancel{3^n}}{\cancel{3^n}\cdot 3}\,\frac{(n^3+3n^2+3n+1)}{n^3} = \frac{1}{3} < 1 \Rightarrow \text{konverguje}$$

Pr 6 $\sum_{n=1}^{\infty} \frac{n!}{2^{n+3}}$

$n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n$

$(n+1)! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n \cdot (n+1)$

$$\lim_{n\to\infty} \frac{\frac{(n+1)!}{2^{n+4}}}{\frac{n!}{2^{n+3}}} = \lim_{n\to\infty} \frac{\cancel{2^n}\cdot \cancel{2^3}}{\cancel{2^n}\cdot 2^4} \cdot \frac{\cancel{n!}\,(n+1)}{\cancel{n!}} = \lim_{n\to\infty} \frac{n+1}{2} = +\infty > 1 \Rightarrow \text{rad divergje}$$

Pr 7. $\sum_{n=1}^{\infty} \frac{2 \cdot n!}{(2n)!}$

$[2(n+1)]! = (2n+2)! = \underbrace{1 \cdot 2 \cdot \ldots \cdot 2n}_{2n!} \cdot (2n+1)(2n+2)$

$$\lim_{n\to\infty} \frac{\frac{2\cdot(n+1)!}{(2(n+1))!}}{\frac{2\cdot n!}{(2n)!}} = \lim_{n\to\infty} \frac{\cancel{(2n)!}\ \cancel{2}\cdot \cancel{n!}\,(n+1)}{\cancel{2}\cdot \cancel{n!}\ \cancel{(2n)!}\,(2n+1)(2n+2)} = \lim_{n\to\infty} \frac{n+1}{4n^2+6n+2} = 0 < 1 \text{ rad konvergje}$$

Pr 8. $\boxed{\sum_{n=5}^{\infty} \frac{(n+1)!\cdot(n+2)!}{(2n)!} \cdot 5^n}$

$$\lim_{n\to\infty} \frac{\frac{(n+1+1)!\cdot(n+3)!}{(2n+2)!}\,5^{n+1}}{\frac{(n+1)!\cdot(n+2)!}{(2n)!}\cdot 5^n} = \lim_{n\to\infty} \frac{\cancel{(n+1)!}(n+2)}{\cancel{(n+1)!}} \cdot \frac{\cancel{(n+2)!}(n+3)}{\cancel{(n+2)!}} \cdot \frac{\cancel{(2n)!}}{\cancel{(2n)!}(2n+1)(2n+2)} \cdot \frac{\cancel{5^n}\cdot 5}{\cancel{5^n}} =$$

$$= \lim_{n\to\infty} \frac{(n+2)(n+3)\cdot 5}{(2n+1)(2n+2)} = \lim_{n\to\infty} \frac{5(n^2+5n+6)}{4n^2+6n+2} = \frac{5}{4} > 1 \Rightarrow \text{rad divergira.}$$