D'Alembertovo kritérium.

Pr1. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

$$\lim_{n\to\infty} \frac{\frac{(n+1)^2}{2^{n+1}}}{\frac{n^2}{2^n}} = \lim_{n\to\infty} \frac{n^2+2n+1}{n^2} \cdot \frac{\cancel{2^n}}{\cancel{2^n}\cdot 2} = \frac{1}{2} < 1 \Rightarrow \text{rad konverguje}$$

Pr2. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{2^{n+3}}$

$$\lim_{n\to\infty} \frac{\frac{(n+1)!}{2^{n+4}}}{\frac{n!}{2^{n+3}}} = \lim_{n\to\infty} \frac{\cancel{2^{n+3}}}{\cancel{2^{n+3}}\cdot 2} \cdot \frac{\cancel{n!}(n+1)}{\cancel{n!}} = \lim_{n\to\infty} \frac{n+1}{2} = \infty > 1 \Rightarrow \text{rad diverg.}$$

Pr3. $\displaystyle\sum_{n=1}^{\infty} \frac{2\cdot n!}{(2n)!}$

$$\lim_{n\to\infty} \frac{\frac{2\cdot(n+1)!}{(2(n+1))!}}{\frac{2\cdot n!}{(2n)!}} = \lim_{n\to\infty} \frac{\cancel{2}\cdot\cancel{n!}\cdot(n+1)}{\cancel{2}\cdot\cancel{n!}} \cdot \frac{\cancel{(2n)!}}{\cancel{(2n)!}(2n+1)(2n+2)} = \lim_{n\to\infty} \frac{n+1}{4n^2+6n+2} =$$

$$= 0 < 1 \Rightarrow \text{rad konverguje}$$

$0! = 1$

Pr4. $\displaystyle\sum_{n=0}^{\infty} \frac{(n+1)!\,(n+3)!}{(2n)!} \cdot 3^n$

$$\lim_{n\to\infty} \frac{\frac{(n+2)!\,(n+4)!}{(2n+2)!}\cdot 3^{n+1}}{\frac{(n+1)!\,(n+3)!}{(2n)!}\cdot 3^n} = \lim_{n\to\infty} \frac{\cancel{(n+1)!}\,(n+2)}{\cancel{(n+1)!}} \cdot \frac{\cancel{(n+3)!}\,(n+4)}{\cancel{(n+3)!}} \cdot \frac{\cancel{3^n}\cdot 3}{\cancel{3^n}} \cdot \frac{\cancel{(2n)!}}{\cancel{(2n)!}\,(2n+1)\cdot(2n+2)} =$$

$$= \lim_{n\to\infty} \frac{n^2+6n+8}{4n^2+6n+2} \cdot \frac{3}{1} = \frac{1}{4}\cdot 3 < 1 \Rightarrow \text{rad konverguje}$$

Neurčitý integrál.

Př 1. $\int x^2+2x+5\,dx = \frac{x^3}{3} + x^2 + 5x + c$

Př 2. $\int \frac{1}{x} + 2\sin x\,dx = \ln|x| + 2(-\cos x) + c = \ln|x| - 2\cos x + c$

Př 3. $\int \frac{2}{1+x^2}\,dx = 2\,\mathrm{arctg}\,x + c$ $\qquad [\mathrm{arctg}\,x]' = \frac{1}{1+x^2}$

Př 4. $\int \sqrt{x} + \frac{1}{\sqrt{x}}\,dx = \int x^{\frac{1}{2}} + x^{-\frac{1}{2}}\,dx = 2.\frac{x^{\frac{3}{2}}}{3} + 2.x^{\frac{1}{2}} + c$

Per partes

Př 5. $\int (x^2+2x+3)\sin x\,dx =^*$ $\qquad \int f'\cdot g\,dx = f\cdot g - \underbrace{\int f\cdot g'\,dx}_{\text{„lehký"}}$

$f' = x^2+2x+3 \qquad f = \frac{x^3}{3}+x^2+3x$

$g = \sin x \qquad g' = \cos x$

$= \left(\frac{x^3}{3}+x^2+3x\right)\cdot\sin x - \int\left(\frac{x^3}{3}+x^2+3x\right).\cos x\,dx$ $\qquad$ Nevýhodné!

—— o ——

$f' = \sin x \qquad f = -\cos x$

$g = x^2+2x+3 \qquad g' = 2x+2$

$\overset{*}{=} -\cos x \cdot (x^2+2x+3) + \int \cos x \cdot (2x+2)\,dx =$ $\qquad f' = \cos x \qquad f = \int f'\,dx = \int \cos x$

další $\qquad f' = \cos x \qquad f = \sin x$

$\qquad g = 2x+2 \qquad g' = 2$

$= -\cos x(x^2+2x+3) + \left[\sin x\,(2x+2) - \int \sin x \cdot 2\,dx\right] =$

$= -\cos x\,(x^2+2x+3) + \sin x\,(2x+2) + 2\cos x + c$

Př6. $\int x^2 \cdot \ln x\,dx =$

$f' = x^2 \qquad f = \frac{x^3}{3}$

$g = \ln x \qquad g' = \frac{1}{x}$

$= \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\cdot\frac{1}{x}\,dx = \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2\,dx = \frac{x^3}{3}\cdot\ln x - \frac{1}{3}\,\frac{x^3}{3} + c$

Př7.

$\int \ln^2 x\,dx = \qquad g^2 = (\ln x)^2 = \ln^2 x$

$\left(\begin{array}{ll} f' = \ln x & f = ? \\ g = \ln x & g' = \frac{1}{x} \end{array}\right) \qquad \begin{array}{ll} f' = 1 & f = x \\ g = \ln^2 x & g' = 2\ln x \cdot \frac{1}{x} \end{array}$

$= x \cdot \ln^2 x - \int x \cdot 2\ln x \cdot \frac{1}{x}\,dx = x \cdot \ln^2 x - 2\int \ln x\,dx =$

$f' = 1 \qquad f = x$

$g = \ln x \qquad g' = \frac{1}{x}$

$$= x\ln^2 x - 2\left[x \cdot \ln x - \int x \cdot \frac{1}{x}\, dx\right] = x\ln^2 x - 2x\ln x + 2x + c$$

Pr 8. $\int x^3 \cdot \operatorname{arctg} x \, dx =$

$f' = x^3 \qquad f = \frac{x^4}{4}$

$g = \operatorname{arctg} x \qquad g' = \frac{1}{1+x^2}$

$$= \frac{x^4}{4} \cdot \operatorname{arctg} x - \int \frac{x^4}{4} \cdot \frac{1}{1+x^2}\, dx = \frac{x^4}{4} \cdot \operatorname{arctg} x - \frac{1}{4}\int \frac{x^4}{1+x^2}\, dx = \frac{x^4}{4} \cdot \operatorname{arctg} x - \frac{1}{4}\int x^2 - 1 + \frac{1}{x^2+1}\, dx =$$

$x^4 : (x^2+1) = x^2 - 1 + \frac{1}{x^2+1}$

$-(x^4 + x^2)$

$-x^2$

$-(-x^2 - 1)$

1

$$= \frac{x^4}{4} \operatorname{arctg} x - \frac{1}{4}\left(\frac{x^3}{3} - x + \operatorname{arctg} x\right) + c$$

Pr 9 $\int \operatorname{arctg} x \, dx =$

$f' = 1 \qquad f = x$

$g = \operatorname{arctg} x \qquad g' = \frac{1}{1+x^2}$

$$\int \frac{\varphi'(x)}{\varphi(x)}\,dx = \ln|\varphi(x)|$$

$$= x\cdot \operatorname{arctg} x - \int x\cdot\frac{1}{1+x^2}\,dx = x\cdot\operatorname{arctg}x - \int\frac{x}{1+x^2}\,dx = x\cdot \operatorname{arctg}x - \frac{1}{2}\int\frac{2x}{1+x^2}\,dx = x\operatorname{arctg}x - \frac{1}{2}\ln(1+x^2)+c$$

$$[\ln(1+x^2)]' = \frac{1}{1+x^2}\,2x$$

$f' = 1 \qquad f = x$

$g = \frac{x}{1+x^2} \qquad g' = \frac{1+x^2-x\cdot 2x}{(1+x^2)^2}$

$\dots\dots - \int\frac{x(1-x^2)}{(1+x^2)^2}\,dx$

$$[\ln\varphi(x)]' = \frac{1}{\varphi(x)}\cdot\varphi'(x)$$

Substituição

10. $\int\cos(3x)\,dx = \frac{1}{3}\int\cos(3x)\cdot 3\,dx = \frac{1}{3}\int\cos y\cdot dy = \frac{1}{3}\sin y + c = \frac{1}{3}\sin(3x)+c$

$y = 3x$

$dy = 3\,dx$

11. $\int\sqrt{7-3x}\,dx = -\frac{1}{3}\int\sqrt{7-3x}\cdot(-3)\,dx = -\frac{1}{3}\int\sqrt{y}\,dy = -\frac{1}{3}\int y^{\frac{1}{2}}\,dy = -\frac{1}{3}\,\frac{y^{\frac{3}{2}}}{\frac{3}{2}} = -\frac{2}{9}\cdot(7-3x)^{\frac{3}{2}}+c$

$y = 7-3x$

$dy = -3\,dx$

12. $\int\frac{x^3+3x-4}{2x-1}\,dx =$

$x^3+3x-4 : (2x-1) = \frac{1}{2}x^2 + \frac{1}{4}x + \frac{13}{8} - \frac{19}{8}\cdot\frac{1}{2x-1}$

$-\left(x^3-\frac{1}{2}x^2\right)$

$$\frac{1}{2}x^2 + 3x - 4$$

$$-\left(\frac{1}{2}x^2 - \frac{1}{4}x\right)$$

$$\frac{13}{4}x - 4$$

$$-\left(\frac{13}{4}x - \frac{13}{8}\right)$$

$$-\frac{19}{8}$$

$$= \frac{1}{2}\frac{x^3}{3} + \frac{1}{4}\frac{x^2}{2} + \frac{13}{8}x - \frac{15}{8}\frac{1}{2}\int \frac{1 \cdot 2}{2x-1}\,dx =$$

$$= \frac{x^3}{6} + \frac{x^2}{8} + \frac{13}{8}x - \frac{15}{16} \cdot \ln|2x-1|\,dx$$