Geometrický rad.

$$\underbrace{a + a\cdot q + a\cdot q^2 + \dots + a q^n + \dots}_{S} = \sum_{n=0}^{\infty} a\cdot q^n$$

$$= a(1+q+\dots q^n+\dots)$$

q – kvocient

Rad konverguje $\iff |q|<1$

Ak —"—, tak jeho súčet $S = a\,\dfrac{1}{1-q}$

Pr 1. $\displaystyle\sum_{n=0}^{\infty} 2\cdot\left(\frac{3}{5}\right)^n = 2\,\frac{1}{1-\frac{3}{5}} = \cancel{2}\,\frac{1}{\frac{\cancel{2}}{5}} = 5$

$a = 2$

$q = \frac{3}{5} < 1$

Pr 1b $\displaystyle\sum_{n=0}^{\infty} 2\cdot\left(\frac{5}{3}\right)^n$ $\qquad |q| = \left|\frac{5}{3}\right| = \frac{5}{3} > 1 \Rightarrow$ rad diverguje.

Pr 2. $\displaystyle\sum_{n=3}^{\infty} 2\cdot\left(\frac{3}{5}\right)^n = 2\cdot\left(\frac{3}{5}\right)^3\cdot\frac{1}{1-\frac{3}{5}} = \cancel{2}\cdot\frac{3^3}{5^{3\,2}}\cdot\frac{\cancel{5}}{\cancel{2}} = \frac{27}{25}$

$a = 2\left(\frac{3}{5}\right)^3 \qquad q = \frac{3}{5} < 1$

pre $n=3$

$$a = 2\left(\frac{3}{5}\right)^3 + 2\left(\frac{3}{5}\right)^4 + 2\left(\frac{3}{5}\right)^5 + \dots =$$

$$= 2\left(\frac{3}{5}\right)^3\left[1 + \underbrace{\frac{3}{5}}_{q} + \left(\frac{3}{5}\right)^2 + \dots\right]$$

Pr 3. $\displaystyle\sum_{n=2}^{\infty} 5\cdot\left(-\frac{1}{4}\right)^n = 5\cdot\frac{1}{16}\cdot\frac{1}{1-(-\frac{1}{4})} = \frac{5}{16}\cdot\frac{4}{5} = \frac{1}{4}$

$a = 5\,\frac{1}{16} \qquad q = -\frac{1}{4} \qquad |q| = \left|-\frac{1}{4}\right| = \frac{1}{4} < 1 \Rightarrow$ konv.

Pr 4. $$\sum_{n=3}^{\infty} \frac{3^{n+1} - (-4)^n}{2^{3n+1}} = \sum_{n=3}^{\infty} \frac{3^{n+1}}{2^{3n+1}} - \sum_{n=3}^{\infty} \frac{(-4)^n}{2^{3n+1}} \overset{*}{=}$$

$$\sum_{n=3}^{\infty} \frac{3^n \cdot 3}{2^{3n} \cdot 2} = \sum_{n=3}^{\infty} \frac{3}{2} \cdot \left(\frac{3}{8}\right)^n = \frac{3}{2} \cdot \left(\frac{3}{8}\right)^3 \cdot \frac{1}{1-\frac{3}{8}} = \frac{81}{2^{10}} \cdot \frac{8}{5} = \frac{81}{1024} \cdot \frac{8}{5}$$

$2^{3n} = (2^3)^n$ $\quad a_1 = \frac{3}{2} \cdot \left(\frac{3}{8}\right)^3 \quad q_1 = \frac{3}{8} < 1$

$$\sum_{n=3}^{\infty} \frac{1}{2} \cdot \frac{(-4)^n}{2^{3n}} = \sum_{n=3}^{\infty} \frac{1}{2} \left(\frac{-4}{8}\right)^n = \frac{-1}{16} \cdot \frac{1}{1-\left(-\frac{1}{2}\right)} = -\frac{1}{16} \cdot \frac{2}{3} = -\frac{1}{8 \cdot 3}$$

$a_2 = \frac{1}{2} \cdot \left(-\frac{1}{2}\right)^3 \quad q_2 = -\frac{1}{2} \quad |q_2| = \left|-\frac{1}{2}\right| = \frac{1}{2} < 1$

$$\overset{*}{=} \frac{81}{128 \cdot 5} + \frac{1}{24}$$

Pr 5. $$\sum_{n=4}^{\infty} \frac{5 \cdot 3^{2n-1} - 3 \cdot (-5)^{n+3}}{7 \cdot 2^{4n+1}} = \sum_{n=4}^{\infty} \frac{5 \cdot 3^{2n-1}}{7 \cdot 2^{4n+1}} - \sum_{n=4}^{\infty} \frac{3 \cdot (-5)^{n+3}}{7 \cdot 2^{4n+1}} \overset{*}{=}$$

$$\sum_{n=4}^{\infty} \frac{5 \cdot 3^{2n-1}}{7 \cdot 2^{4n+1}} = \sum_{n=4}^{\infty} \frac{5 \cdot (3^2)^n \cdot 3^{-1}}{7 \cdot (2^4)^n \cdot 2^1} = \sum_{n=4}^{\infty} \frac{5}{7 \cdot 2 \cdot 3} \cdot \left(\frac{9}{16}\right)^n = \frac{5}{7 \cdot 2 \cdot 3} \left(\frac{9}{16}\right)^4 \cdot \frac{1}{1-\frac{9}{16}} = \frac{5}{7 \cdot 6} \cdot \left(\frac{9}{16}\right)^4 \cdot \frac{16}{7} =$$

$a = \frac{5}{7 \cdot 2 \cdot 3} \cdot \left(\frac{9}{16}\right)^4 \quad q = \frac{9}{16}$

$$= \frac{5}{7^2 \cdot 6} \cdot \frac{9^4}{16^3}$$

$$\sum_{n=4}^{\infty} \frac{3\cdot(-5)^{n+3}}{7\cdot 2^{4n+1}} = \sum_{n=4}^{\infty} \frac{3\cdot(-5)^3}{7\cdot 2}\cdot\left(\frac{-5}{16}\right)^n = \frac{3\cdot(-5)^7}{14\cdot 16^4}\cdot\frac{1}{1-\left(-\frac{5}{16}\right)} = \frac{\cancel{3}\cdot(-5)^7}{14\cdot 16^{\cancel{4}\,3}}\cdot\frac{\cancel{16}}{\cancel{21}\,7}$$

$$a = \frac{3\cdot(-5)^3}{14}\cdot\left(\frac{-5}{16}\right)^4 \qquad q = -\frac{5}{16} \qquad |q| < 1$$

$$=^* \frac{5}{7^2\cdot 6}\cdot\frac{9^4}{16^3} - \frac{(-5)^7}{14\cdot 16^3\cdot 7}$$

Porovnávacie kritérium

$$\sum_{n=n_0}^{\infty} a_n \,,\quad \sum_{n=n_0}^{\infty} b_n \qquad\qquad 0 \le a_n \quad 0 \le b_n$$

Porovnanie $a_n \le b_n$ od istého n_1 (do nekonečna)

Ak vieme, že $\sum b_n$ konverguje, tak aj $\sum a_n$ konverguje

—||— $\sum a_n$ diverguje, tak aj $\sum b_n$ diverguje.

Pr 6 $\displaystyle\sum_{n=0}^{\infty} \frac{n+1}{n^3+3n+4}$ Známy rad $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^3} = \sum_{n=1}^{\infty} \frac{1}{n^2}$ – konverguje

Exponenty v čit. n^1

v men. n^3

$$a_n = \frac{n+1}{n^3+3n+4} \leq \frac{n+n}{n^3} = \frac{2n}{n^3} = 2\frac{1}{n^2} \quad \text{od } n=1$$

$$\sum_{n=1}^{\infty} 2\frac{1}{n^2} = 2 \cdot \sum_{n=1}^{\infty} \frac{1}{n^2} \quad \text{konverguje}$$

$$\Rightarrow \sum_{n=0}^{\infty} \frac{n+1}{n^3+3n+4} \quad \text{tiež konverguje}$$

$$\shortparallel$$

$$\frac{1}{4} + \sum_{n=1}^{\infty} \frac{n+1}{n^3+3n+4}$$

$$\boxed{\sum_{n=n_1}^{\infty} a_n \qquad \sum_{n=n_2}^{\infty} b_n \quad \text{a platí} \quad \lim_{n\to\infty} \frac{a_n}{b_n} = p \qquad 0<p<\infty \quad \text{(p kladné číslo)} \quad \text{tak oba rady sa rovnako chovajú}}$$

Pr 7. $\displaystyle\sum_{n=0}^{\infty} \frac{n-1}{n^2-2n+5}$

S akým radom ho porovnáme?

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ – diverguje

$$\lim_{n\to\infty} \frac{\frac{n-1}{n^2-2n+5}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^2-n}{n^2-2n+5} = \frac{1}{1} = 1 = p$$

$0<p<\infty$ $\Big\}$ $\Rightarrow$ Rad

$$\sum_{n=0}^{\infty} \frac{n-1}{n^2-2n+5} \quad \text{tiež diverguje}$$

Pr 8.

$$\sum_{n=0}^{\infty} \frac{n^2-3n+4}{\sqrt{n^7-2n^5+5n^3-1}}$$

S akým radom ho porovnáme?

$\sum \frac{1}{n^2}$

$\cancel{\sum \frac{1}{\sqrt{n}}}$

$\cancel{\sum \frac{1}{n^{\frac{5}{2}}}}$

$\cancel{\sum \frac{1}{n\sqrt{n}}}$

Exp v čit. 2

v men. $\frac{7}{2}$ $\qquad \frac{7}{2}-2=\frac{3}{2}$

$$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}} \quad - \text{konverguje}$$

$$\boxed{\sum_{n=1}^{\infty} \frac{1}{n^{\alpha}} \quad \begin{array}{ll} \text{konverguje keď} & 1<\alpha \\ \text{diverguje keď} & \alpha \le 1 \end{array}}$$

$$\lim_{n\to\infty} \frac{\frac{n^2-3n+4}{\sqrt{n^7-2n^5+5n^3-1}}}{\frac{1}{n^{\frac{3}{2}}}} = \lim_{n\to\infty} \frac{n^{\frac{7}{2}}-3n^{\frac{5}{2}}+4n^{\frac{3}{2}}}{\sqrt{n^7}\sqrt{\left(1-\frac{2}{n^2}+5\frac{1}{n^4}-\frac{1}{n^7}\right)}} = \frac{1}{1} = 1$$

$$0<1<\infty \Rightarrow$$

Rad $\sum_{n=0}^{\infty} \frac{n^2-3n+4}{\sqrt{n^7-2n^5+5n^3-1}}$ tiež konverguje.

Prečo je porovnanie s radom $\sum \frac{1}{n^2}$ zlé?

$$\lim_{n\to\infty} \frac{\frac{n^2-3n+4}{\sqrt{n^7-2n^5+5n^3-1}}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^4-3n^3+4n^2}{\sqrt{n^7}\sqrt{1-2\frac{1}{n^2}+5\frac{1}{n^4}-\frac{1}{n^7}}} = \infty$$

exp $4 = \frac{8}{2}$

exp. $\frac{7}{2}$

Pr 9 $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$ $\qquad \sum_{n=1}^{\infty} \frac{1}{n^2}$ – konvergujе

$$\lim_{n\to\infty} \frac{\sin\left(\frac{1}{n^2}\right)}{\frac{1}{n^2}} = \lim_{y\to 0} \frac{\sin y}{y} = 1 \qquad 0 < 1 < \infty \implies$$

$\frac{1}{n^2} - y$

Rad $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$ konvergujе

Pr 9b $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$ $\qquad \sum_{n=1}^{\infty} \frac{1}{n}$ – divergujе $\implies$