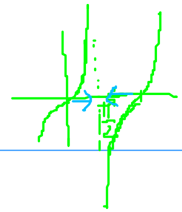
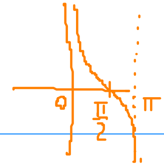
# L'Hospitalovo pravilo





Pr 1. $$\lim_{x\to 2-} \underbrace{\frac{x^2-4}{x^2}}_{\to 0} \cdot \underbrace{\mathrm{tg}\left(\frac{\pi}{4}\cdot x\right)}_{\to +\infty} = \lim_{x\to 2-}\frac{1}{x^2}\cdot \lim_{x\to 2-}(x^2-4)\cdot \mathrm{tg}\left(\frac{\pi}{4}\cdot x\right) = \frac{1}{4}\cdot \lim_{x\to 2-}\frac{\overset{\to 0}{x^2-4}}{\underset{\to 0}{\mathrm{ctg}\left(\frac{\pi}{4}\cdot x\right)}} \overset{L'H}{=} \frac{1}{4}\lim_{x\to 2-}\frac{2x}{-\frac{1}{\sin^2\left(\frac{\pi}{4}\cdot x\right)}\cdot\frac{\pi}{4}} =$$

$0 \cdot +\infty$

$$= \frac{1}{\cancel{4}}\cdot\frac{\cancel{4}}{-\frac{1}{1}\cdot\frac{\pi}{4}} = -\frac{4}{\pi}$$

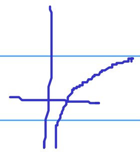
$\cos^2 x - 1 = -\sin^2 x$

Pr 2. $$\lim_{x\to 0+} \underset{\to +\infty}{\frac{\cos x}{x}} - \underset{\to +\infty}{\frac{1}{\sin x}} = \lim_{x\to 0+}\frac{\cos x\cdot\sin x - x}{\underset{\to 0}{x\cdot\sin x}} \overset{L'H}{=} \lim_{x\to 0+}\frac{-\sin x\cdot\sin x + \cos x\cdot\cos x - 1}{\underset{\to 0}{\sin x} + \underset{\to 0}{x\cos x}} = \lim_{x\to 0+}\frac{-\sin^2 x + \cos^2 x - 1}{\sin x + x\cdot\cos x} =$$

$+\infty - +\infty$

$0 + 1 - 1 = 0$

$$= \lim_{x\to 0+}\frac{-2\sin^2 x}{\sin x + x\cos x} \overset{L'H}{=} \lim_{x\to 0+}\frac{-2\cdot 2\sin x\cdot\cos x}{\cos x + \cos x + x(-\sin x)} = \frac{0}{2} = 0 = \lim_{x\to 0-}\frac{\cos x}{x} - \frac{1}{\sin x}$$



Pr 3. $$\lim_{x\to 1+} \underset{\to +\infty}{\frac{x}{x-1}} - \underset{\to +\infty}{\frac{1}{\ln x}} = \lim_{x\to 1+}\frac{x\cdot\ln x - (x-1)}{(x-1)\cdot\ln x} \overset{L'H}{=} \lim_{x\to 1+}\frac{\ln x + \cancel{x}\frac{1}{\cancel{x}} - 1}{\ln x + (x-1)\frac{1}{x}} = \lim_{x\to 1+}\frac{\ln x}{\ln x + 1 - \frac{1}{x}} \overset{L'H}{=}$$

$+\infty - +\infty$

$$\lim_{x\to 1+}\frac{\frac{1}{x}}{\frac{1}{x}+\frac{1}{x^2}} = \frac{1}{1+1} = \frac{1}{2}$$

$$\left[f(x)\right]^{g(x)} = \left[e^{\ln f(x)}\right]^{g(x)} = e^{g(x)\cdot \ln f(x)} \qquad f(x) = e^{\ln f(x)}$$

Př 4

$$\lim_{x\to 0+} \left(\frac{1}{x}\right)^{x} = \lim_{x\to 0+} e^{x\cdot \ln \frac{1}{x}} = e^{0} = 1$$

Pomocná limita

$$\lim_{x\to 0+} x\cdot \ln\frac{1}{x} = \lim_{x\to 0+} \frac{\overset{\nearrow \infty}{\ln \frac{1}{x}}}{\underset{\searrow +\infty}{\frac{1}{x}}} \overset{L'H}{=} \lim_{x\to 0+} \frac{\frac{1}{\frac{1}{x}}\cdot \cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{-\frac{1}{x^2}}} = \lim_{x\to 0+} x = 0$$

Př 5.

$$\lim_{x\to\infty} \left(1+\frac{3}{x}\right)^{x} = \lim_{x\to\infty} e^{x\cdot \ln\left(1+\frac{3}{x}\right)} = e^{3}$$

$$\lim_{x\to\infty} x\cdot \ln\left(1+\frac{3}{x}\right) = \lim_{x\to\infty} \frac{\ln\left[1+\frac{3}{x}\right]}{\frac{1}{x}} \overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{1+\frac{3}{x}}\cdot 3\cdot \cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{-\frac{1}{x^2}}} = \frac{3}{1} = 3$$

Př 6.

$$\lim_{x\to\infty} \left(1+\frac{k}{x}\right)^{x} = e^{k} \qquad k \neq 0$$

# Nekonečné rady.

$$\underbrace{\underbrace{\underbrace{\underbrace{a_0}_{S_0} + a_1}_{S_1} + a_2}_{S_2} + \dots + a_n}_{S_n} + \dots = \sum_{n=0}^{\infty} a_n$$

$S_0, S_1, S_2, \dots, S_n, \dots \longrightarrow S$

$\lim_{n\to\infty} S_n = S$

ak $S$ je číslo tak $\sum_{n=0}^{\infty} a_n = S$

$S = \pm\infty$ tak rad $\sum_{n=0}^{\infty} a_n$ diverguje do $\pm\infty$

$S$ – neexistuje, tak rad diverguje

Pr 1. $\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$ nájdime jeho súčet

Príprava $\frac{1}{n^2+2n} = \frac{A}{n} + \frac{B}{n+2} = \frac{A\cdot(n+2)+Bn}{n\cdot(n+2)} \Rightarrow 1 = A(n+2)+Bn$ *

$n^2+2n = n\cdot(n+2)$

1. Prístup $0\cdot n + 1 = (A+B)n + 2A$

$n: \; 0 = A+B$

$n^0: \; 1 = 2A$

$\Rightarrow A = \frac{1}{2}, \; B = -\frac{1}{2}$

2. Prístup * Platí aj pre $n=0$

$1 = A\cdot 2 + B\cdot 0 \Rightarrow A = \frac{1}{2}$

aj pre $n=-2$ $\quad 1 = A\cdot 0 + B\cdot(-2) \Rightarrow B = -\frac{1}{2}$

$$\frac{1}{n^2+2n} = \frac{1/2}{n} - \frac{1/2}{n+2}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2+2n} = \left(\frac{1/2}{1} - \frac{1/2}{3}\right) + \left(\frac{1/2}{2} - \frac{1/2}{4}\right) + \dots + \left(\frac{1/2}{n} - \frac{1/2}{n+2}\right) + \dots$$

$$S_n = \left(\frac{1/2}{1} - \cancel{\frac{1/2}{3}}\right) + \left(\frac{1/2}{2} - \cancel{\frac{1/2}{4}}\right) + \left(\cancel{\frac{1/2}{3}} - \cancel{\frac{1/2}{5}}\right) + \left(\cancel{\frac{1/2}{4}} - \cancel{\frac{1/2}{6}}\right) + \dots + \left(\cancel{\frac{1/2}{n-1}} - \frac{1/2}{n+1}\right) + \left(\cancel{\frac{1/2}{n}} - \frac{1/2}{n+2}\right) =$$

$$= \frac{1}{2} + \frac{1}{4} - \frac{1/2}{n+1} - \frac{1/2}{n+2}$$ $n$-tý čiastočný súčet

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1}{2} + \frac{1}{4} - \frac{1/2}{n+1} - \frac{1/2}{n+2} = \underline{\underline{\frac{3}{4}}}$$

Pr2. $\displaystyle\sum_{n=2}^{\infty} \frac{3}{n^2+n-2}$

$$a_n = \frac{3}{n^2+n-2} = \frac{A}{n-1} + \frac{B}{n+2} \implies 3 = A(n+2) + B(n-1)$$

Pre $n=1$

$$3 = A \cdot 3 \qquad A = 1$$

$$n^2+n-2 = (n-1)(n+2) = n^2 - n + 2n - 2 \checkmark$$

$n_1 = 1 \quad n_2 = -2$

Pre $n=-2$

$$3 = B \cdot -3 \qquad B = -1$$

$$a_n = \frac{1}{n-1} - \frac{1}{n+2}$$

$$\sum_{n=2}^{\infty} \frac{3}{n^2+n-2} = \frac{1}{1} - \frac{1}{4} + \frac{1}{2} - \frac{1}{5} + \frac{1}{3} - \frac{1}{6} + \frac{1}{4} - \frac{1}{7} + \frac{1}{5} - \frac{1}{8} + \dots + \frac{1}{n-1} - \frac{1}{n+2} + \dots$$

$$S_n = \left(\underbrace{\frac{1}{1}} - \cancel{\frac{1}{4}}\right) + \left(\underbrace{\frac{1}{2}} - \cancel{\frac{1}{5}}\right) + \left(\underbrace{\frac{1}{3}} - \cancel{\frac{1}{6}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{7}}\right) + \cancel{\frac{1}{5}} - \frac{1}{8} + \cancel{\dots} + \cancel{\frac{1}{n-1}} - \frac{1}{n+2}$$

$$S_n = 1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{n} - \frac{1}{n+1} - \frac{1}{n+2}$$

$$S = \lim_{n \to \infty} S_n = 1 + \frac{1}{2} + \frac{1}{3} - 0 - 0 - 0 = \frac{6+3+2}{6} = \frac{11}{6}$$

Pr3. $\displaystyle\sum_{n=3}^{\infty} \frac{8}{n^3-4n}$

$$a_n = \frac{8}{n^3-4n} = \frac{A}{n-2} + \frac{B}{n} + \frac{C}{n+2}$$

$$8 = A\,n(n+2) + B(n-2)(n+2) + C(n-2)n$$

$$n^3-4n = n(n^2-4) = n(n-2)(n+2)$$

$n_1=0 \quad n_2=2 \quad n_3=-2$

$n=0$

$8=-4B \qquad B=-2$

$n=2$

$8 = A\cdot 8 \qquad A=1$

$n-2$

$8 = C\cdot 8 \qquad C=1$

$$a_n = \frac{1}{n-2} - \frac{2}{n} + \frac{1}{n+2}$$

$a_3$ $a_4$ $a_5$ $a_6$

$$S_n = \left(1 - \frac{2}{3} + \cancel{\frac{1}{5}}\right) + \left(\frac{1}{2} - \frac{2}{4} + \cancel{\frac{1}{6}}\right) + \left(\frac{1}{3} - \cancel{\frac{2}{5}} + \cancel{\frac{1}{7}}\right) + \left(\frac{1}{4} - \cancel{\frac{2}{6}} + \cancel{\frac{1}{8}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{2}{7}} + \cancel{\frac{1}{9}}\right) + \cancel{\ldots\ldots}$$

$$\cancel{\ldots} + \left(\cancel{\frac{1}{n-4}} - \cancel{\frac{2}{n-2}} + \frac{1}{n}\right) + \left(\cancel{\frac{1}{n-3}} - \frac{\cancel{2}\;1}{n-1} + \frac{1}{n+1}\right) + \left(\cancel{\frac{1}{n-2}} - \frac{2}{n} + \frac{1}{n+2}\right)$$

$$S_n = 1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} \quad - \frac{1}{n} - \frac{1}{n-1} + \frac{1}{n+1} + \frac{1}{n+2}$$

$$\lim_{n\to\infty} S_n = \frac{12 - 4 + 6 - 3}{12} - 0 = \frac{11}{12}$$