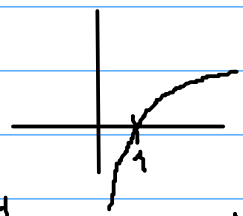
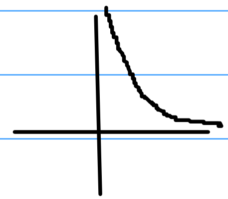
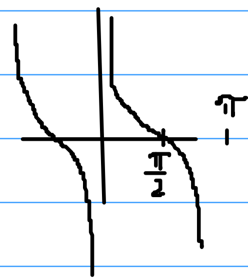
l'Hospitalovo pravidlo

Súčin „$0\cdot\infty$" $\longrightarrow$ podiel



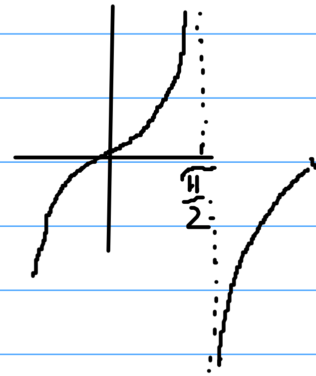




Príklad 1.

$$\lim_{x\to0+} x\cdot\ln x = \lim_{x\to0+}\frac{\ln x}{\frac{1}{x}} \overset{L'H}{=} \lim_{x\to0+}\frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to0+}\frac{-x^{\cancel{2}}}{\cancel{x}} = 0$$

„$\frac{-\infty}{+\infty}$"

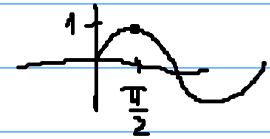


Príklad 2.

$$\lim_{x\to0}(e^x-1)\cdot\cotg x = \lim_{x\to0}\frac{e^x-1}{\tg x} \overset{L'H}{=} \lim_{x\to0}\frac{e^x}{\frac{1}{\cos^2 x}} = \frac{1}{1} = 1$$

„$\frac{0}{0}$"

$$\lim_{x\to0} e^x\cdot\frac{\cos^2 x}{1} = 1$$



Príklad 3.

$$\lim_{x\to2}\frac{x^2-4}{x^2}\cdot\tg\left(\frac{\pi}{4}x\right) = \lim_{x\to2}\frac{1}{x^2}\cdot\lim_{x\to2}\frac{x^2-4}{\cotg\left(\frac{\pi}{4}x\right)} = \frac{1}{4}\cdot\lim_{x\to2}\frac{2x}{-\frac{1}{\sin^2\left(\frac{\pi}{4}x\right)}\cdot\frac{\pi}{4}} = \frac{1}{\cancel{4}}\cdot\frac{\cancel{4}}{1\cdot\frac{\pi}{4}} = \frac{4}{\pi}$$

„$\frac{0}{0}$"

Rozdiel „$\infty-\infty$" $\longrightarrow$ podiel

$\sin 2\alpha = 2\sin\alpha\cos\alpha$ $\qquad -1+\cos^2 x = -\sin^2 x$

Pr. 4

$$\lim_{x\to 0}\left(\frac{\cos x}{x}-\frac{1}{\sin x}\right)=\lim_{x\to 0}\frac{\sin x\cos x - x}{x\cdot\sin x}\overset{L'H}{=}\lim_{x\to 0}\frac{\cos^2 x-\sin^2 x-1}{\sin x + x\cos x}=\lim_{x\to 0}\frac{-2\sin^2 x}{\sin x + x\cos x}=$$

sprava $\infty - \infty$ $\qquad$ "$\frac{0}{0}$" $\qquad$ "$\frac{0}{0}$"

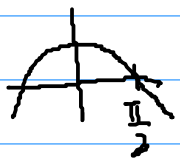
zľava $-\infty-(-\infty)$

$$\overset{L'H}{=}\lim_{x\to 0}\frac{-2\cdot 2\cdot\sin x\cdot\cos x}{\cos x+\cos x - x\sin x}=\frac{0}{2}=0$$

Pr5

$$\lim_{x\to\frac{\pi}{2}-}\left(\operatorname{tg} x-\frac{1}{\cos x}\right)=\lim_{x\to\frac{\pi}{2}-}\frac{\sin x-1}{\cos x}\overset{L'H}{=}\lim_{x\to\frac{\pi}{2}-}\frac{\cos x}{-\sin x}=\frac{0}{1}=0$$

$\infty - \infty$ $\qquad$ "$\frac{0}{0}$"



Pr6.

$$\lim_{x\to 1+}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right)=\lim_{x\to 1+}\frac{x\cdot\ln x-(x-1)}{(x-1)\cdot\ln x}\overset{L'H}{=}\lim_{x\to 1+}\frac{\ln x+\cancel{x\cdot\frac{1}{x}}-\cancel{1}}{\ln x+(x-1)\cdot\frac{1}{x}}\overset{L'H}{=}\lim_{x\to 1+}\frac{\frac{1}{x}}{\frac{1}{x}+\frac{x-(x-1)}{x^2}}=\frac{1}{1+1}$$

$\infty - \infty$ $\qquad$ "$\frac{0}{0}$" $\qquad$ "$\frac{0}{0}$" $\qquad \frac{x-1}{x}$

$$=\frac{1}{2}$$

Typ "1^{∞}"

$$\boxed{\begin{array}{l} \underline{a>0} \\ a = e^{\ln a} \end{array}}$$

Př 7. $$\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^x = \lim_{x\to\infty}\left[e^{\ln\left(1+\frac{3}{x}\right)}\right]^x = \lim_{x\to\infty} e^{x\ln\left(1+\frac{3}{x}\right)} = \underline{\underline{e^3}}$$

Př. $$\lim_{x\to\infty} x\cdot\ln\left(1+\frac{3}{x}\right) = \lim_{x\to\infty}\frac{\ln\left(1+\frac{3}{x}\right)}{\frac{1}{x}} \overset{L'H}{=} \lim_{x\to\infty}\frac{\frac{1}{1+\frac{3}{x}}\cdot 3\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{-\frac{1}{x^2}}} = \underline{3}$$

"$\infty\cdot 0$" "$\frac{0}{0}$" $\frac{3}{x}\to 0$

Nekonečné řady

Součet $$\sum_{n=0}^{\infty} a_n = \underbrace{a_0}_{S_0} + a_1 + a_2 + \dots + a_n + \dots = S$$

$$\underbrace{a_0 + a_1}_{S_1}, \quad \underbrace{a_0 + a_1 + a_2}_{S_2}, \quad \underbrace{a_0 + a_1 + a_2 + \dots + a_n}_{S_n}$$

$$\lim_{n\to\infty} S_n = S$$

Př 1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$

$$a_n = \frac{1}{n^2+2n} = \frac{A}{n} + \frac{B}{n+2} = \frac{1/2}{n} - \frac{1/2}{n+2}$$

$$n^2+2n = n(n+2)$$

$$1 = A(n+2) + Bn$$

Dosaď $n=0$ $\quad 1 = A \cdot 2 \quad\quad A = \frac{1}{2}$

$n=-2$ $\quad 1 = -2B \quad\quad B = -\frac{1}{2}$

$$S_1 = \frac{1/2}{1} - \frac{1/2}{3}$$

$$S_2 = \frac{1/2}{1} - \frac{1/2}{3} + \frac{1/2}{2} - \frac{1/2}{4}$$

$$S_N = \frac{1/2}{1} - \cancel{\frac{1/2}{3}} + \frac{1/2}{2} - \cancel{\frac{1/2}{4}} + \cancel{\frac{1/2}{3}} - \cancel{\frac{1/2}{5}} + \cancel{\frac{1/2}{4}} - \cancel{\frac{1/2}{6}} + \cancel{\ldots} \cancel{\frac{1/2}{N-1}} - \frac{1/2}{N+1} + \cancel{\frac{1/2}{N}} - \frac{1/2}{N+2} =$$

$$S_N = \frac{1/2}{1} + \frac{1/2}{2} - \frac{1/2}{N+1} - \frac{1/2}{N+2}$$

$$\lim_{N\to\infty} S_n = \frac{1}{2} + \frac{1}{4} - 0 - 0 = \frac{3}{4}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2+2n} = \frac{3}{4}$$

Pr 2. $\sum_{n=3}^{\infty} \frac{1}{n^2-4}$

$$\frac{1}{n^2-4} = \frac{A}{n-2} - \frac{B}{n+2} = \frac{1/4}{n-2} - \frac{1/4}{n+2}$$

$$n^2-4=(n-2)(n+2)$$

$$1 = A(n+2) - B(n-2)$$

Dosadíme

$n=-2 \quad 1 = +B\cdot 4 \quad B=\frac{1}{4}$

$n=2 \quad 1 = 4A \quad A=\frac{1}{4}$

$$S_N = \frac{1/4}{1} - \cancel{\frac{1/4}{5}} + \frac{1/4}{2} - \cancel{\frac{1/4}{6}} + \frac{1/4}{3} - \cancel{\frac{1/4}{7}} + \frac{1/4}{4} - \cancel{\frac{1/4}{8}} + \cancel{\frac{1/4}{5}} - \cancel{\frac{1/4}{9}} + \cancel{\frac{1/4}{6}} - \cancel{\frac{1/4}{10}} + \dots +$$

$$\dots + \left(\cancel{\frac{1/4}{N-5}} - \frac{1/4}{N-1}\right) + \left(\cancel{\frac{1/4}{N-4}} - \frac{1/4}{N}\right) + \left(\cancel{\frac{1/4}{N-3}} - \frac{1/4}{N+1}\right) + \left(\cancel{\frac{1/4}{N-2}} - \frac{1/4}{N+2}\right) =$$

$$S_N = \frac{1/4}{1} + \frac{1/4}{2} + \frac{1/4}{3} + \frac{1/4}{4} - \frac{1/4}{N-1} - \frac{1/4}{N} - \frac{1/4}{N+1} - \frac{1/4}{N+2}$$

$$\lim_{N\to\infty} S_N = \frac{1}{4}\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right) = \frac{1}{4}\,\frac{12+6+4+3}{12} = \frac{25}{48}$$