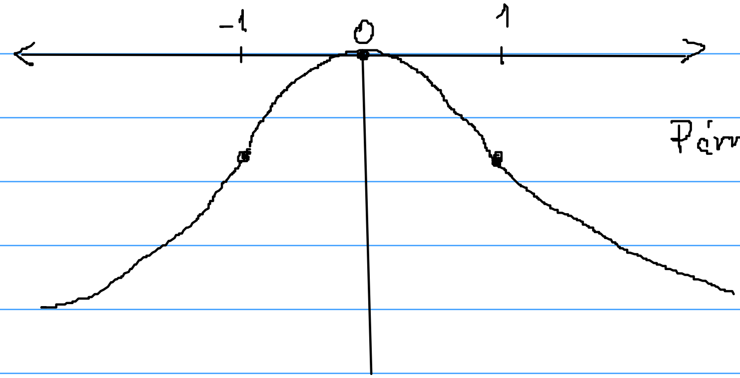
konvexnosť

Pr 7 $\quad f(x) = \ln\left(\frac{1}{1+x^2}\right) \qquad D_f = \mathbb{R}$

$$\left[(1+x^2)^{-1}\right]' = -1\,(1+x^2)^{-2}\cdot 2x = -\frac{2x}{(1+x^2)^2}$$

$$f'(x) = \frac{1}{\frac{1}{1+x^2}}\cdot\left(-\frac{2x}{(1+x^2)^2}\right) = -\frac{2x}{1+x^2}$$

$$f''(x) = -\frac{2(1+x^2) - 2x(2x)}{(1+x^2)^2} = -\frac{2+2x^2-4x^2}{(1+x^2)^2} = \frac{2x^2-2}{(1+x^2)^2}$$



Párna

$$f''(x) = 0 \iff 2x^2 - 2 = 0 \iff x^2 = 1 = \quad x_1 = -1 \ \vee \ x_2 = +1$$

f'': $+$ | -1 | $-$ | 1 | $+$

f: $\cup$ | $\cap$ | $\cup$

konvexná na $(-\infty, -1\rangle$ a $\langle 1, \infty)$

konkávna na $\langle -1, 1\rangle$.

inflexné body sú $x_1 = -1$ $\quad x_2 = 1$

Monotónnosť, L.E., konvexnosť

1. $f(x) = x^2 \cdot e^{\frac{1}{x}}$ $\qquad D_f = R - \{0\}$

$$f'(x) = 2x \cdot e^{\frac{1}{x}} + x^2 \cdot e^{\frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right) = \boxed{e^{\frac{1}{x}} \cdot (2x-1)}$$

$e^{\frac{1}{x}} > 0$

$x_0 = \frac{1}{2}$ je stac. bod

f'	$-$		$-$		$+$
f	$\searrow$	0	$\searrow$	$\frac{1}{2}$	$\nearrow$

Klesajúca na $(-\infty, 0)$ a na $(0, \frac{1}{2}\rangle$

Rastúca na $\langle \frac{1}{2}, \infty)$

Bod $x_0 = \frac{1}{2}$ je Bod OLMIN $\qquad f(\frac{1}{2}) = \frac{1}{4} \cdot e^2$

$$f''(x) = e^{\frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right) \cdot (2x-1) + e^{\frac{1}{x}} \cdot 2 = e^{\frac{1}{x}} \left(\frac{-2x+1+2x^2}{x^2}\right)$$

>0 $\qquad x^2 > 0$ na D_f

$f''(x) = 0 \iff 2x^2 - 2x + 1 = 0$

$$x_{1,2} = \frac{2 \pm \sqrt{4 - 4 \cdot 2(+1)}}{4}$$

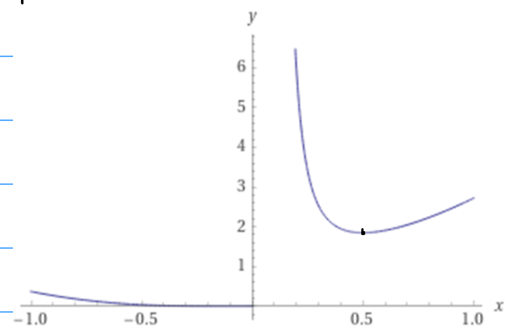
$4 - 8 = -4 < 0 \implies$ nemá reálne korene

$2x^2 - 2x + 1 > 0$

$f''(x) > 0$ na D_f

f''	$+$		$+$
f	$\cup$	0	$\cup$

f je konvexná na $(-\infty, 0)$ a na $(0, \infty)$



2. $f(x) = \frac{2x-1}{(x-1)^2}$ $\qquad D_f = R - \{1\}$

$$f'(x) = \frac{2(x-1)^{\cancel{2}\,1} - (2x-1)\cdot 2\cancel{(x-1)}\cdot 1}{(x-1)^{\cancel{4}\,3}} = \frac{2x-2-(4x-2)}{(x-1)^3} = \boxed{\frac{-2x}{(x-1)^3}}$$

St. bod: $x_0 = 0$

f'	$-$		$+$		$-$
f	$\searrow$	0	$\nearrow$	1	$\searrow$

$$f'\left(\tfrac{1}{2}\right) = \frac{-1}{\left(-\frac{1}{2}\right)^3} = +8 > 0$$

Rastúca na $\langle 0, 1)$

klesajúca na $(-\infty, 0\rangle$ a $(1, \infty)$

Bod $x_0 = 0$ je bod OLMin $\qquad f(0) = -1$

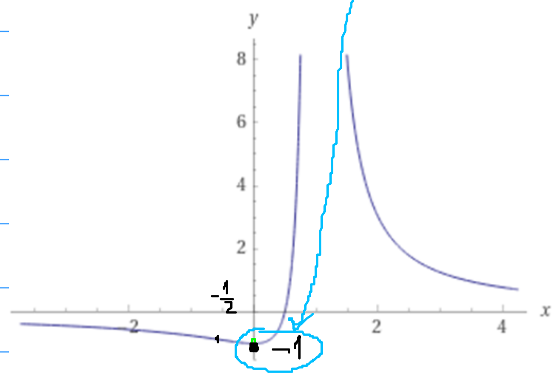
$$f''(x) = \frac{-2(x-1)^{\cancel{3}\,1} + 2x\cdot 3\cdot \cancel{(x-1)^2}}{(x-1)^{\cancel{6}\,4}} = \frac{-2x+2+6x}{(x-1)^4} = \frac{4x+2}{(x-1)^4}$$

$f''(x) = 0 \iff x_1 = -\frac{1}{2}$ je inflexný bod

f''	$-$		$+$		$+$
f	$\cap$	$-\frac{1}{2}$	$\cup$	1	$\cup$

Konvexná na $\langle -\frac{1}{2}, 1)$ a na $(1, \infty)$

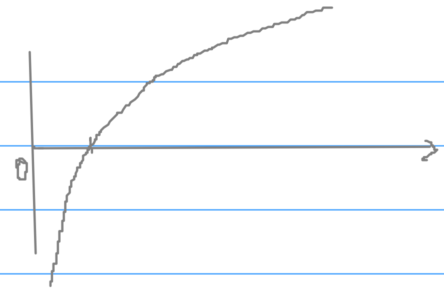
konkávna na $(-\infty, -\frac{1}{2}\rangle$



(x from −3.7 to 4.2)

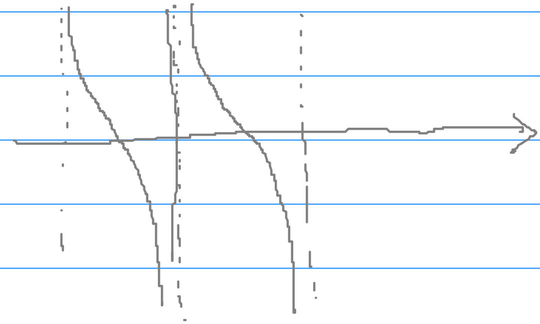
L'Hospitalovo pravidlo

$$\lim_{x\to x_0} \frac{\overset{\pm\infty \;\; 0}{f(x)}}{\underset{\pm\infty \;\; 0}{g(x)}} = \lim_{x\to x_0} \frac{f'(x)}{g'(x)} = L$$



Pr1. $$\lim_{x\to 0} \frac{\overset{1-1=0}{e^{3x}-1}}{\underset{0}{x}} \overset{L'H}{=} \lim_{x\to 0} \frac{e^{3x}\cdot 3}{1} = 3$$

Pr2. $$\lim_{x\to 0+} \frac{\overset{-\infty}{\ln x}}{\underset{+\infty}{\cotg x}} \overset{L'H}{=} \lim_{x\to 0+} \frac{\frac{1}{x}}{-\frac{1}{\sin^2 x}} = \lim_{x\to 0+} -\frac{\sin^2 x}{x} =$$



$$= -\lim_{x\to 0+} \underbrace{\frac{\sin x}{x}}_{1} \cdot \underbrace{\sin x}_{0} = 0$$

neex.

0 ||||||||

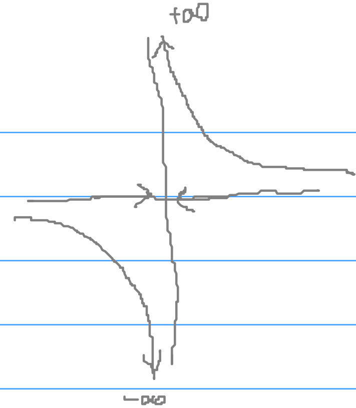
Pr3. $$\lim_{x\to 0} \frac{\overset{0}{x-\sin x}}{\underset{0}{x\cdot \sin^2 x}} \overset{L'H}{=} \lim_{x\to 0} \frac{1-\cos x}{1\cdot \sin^2 x + x\cdot 2\sin x\cdot \cos x} = \lim_{x\to 0} \frac{\overset{0}{1-\cos x}}{\underset{0}{\sin^2 x + x\sin 2x}} \overset{L'H}{=} \lim_{x\to 0} \frac{\sin x}{2\sin x\cdot \cos x + \sin 2x + x\cos 2x\cdot 2}$$

$2\sin\alpha\cos\alpha = \sin 2\alpha$

$$= \lim_{x\to 0} \frac{\overset{0}{\sin x}}{\underbrace{2\sin 2x}_{0} + \underbrace{2x\cos 2x}_{0}} \overset{L'H}{=} \lim_{x\to 0} \frac{\cos x}{2\cos 2x\cdot 2 + 2\cos 2x + 2x(-\sin 2x)\cdot 2} = \lim_{x\to 0} \frac{\cos x}{6\cos 2x - 4x\sin 2x} =$$

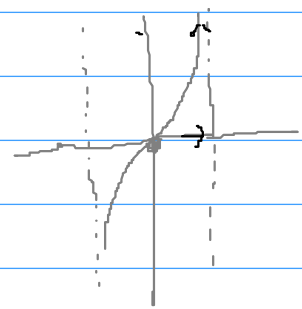
$$= \frac{1}{6-0} = \frac{1}{6}$$

Pr4. $\lim\limits_{x\to 0+} x\cdot \ln x = \lim\limits_{x\to 0+} \dfrac{\overset{-\infty}{\ln x}}{\underset{+\infty}{\frac{1}{x}}} \overset{L'H}{=}$

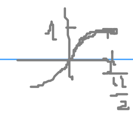


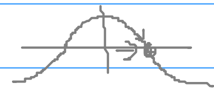
$= \lim\limits_{x\to 0+} \dfrac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim\limits_{x\to 0+} - \dfrac{x^{\cancel{2}}}{\cancel{x}} = 0$

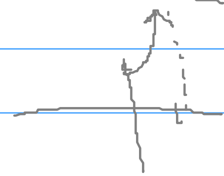
Pr5. $\lim\limits_{x\to 0} (e^x - 1)\cdot \text{cotg}\, 2x = \lim\limits_{x\to 0} \dfrac{\overset{1-1=0}{e^x - 1}}{\underset{0}{\text{tg}\, 2x}} \overset{L'H}{=} \lim\limits_{x\to 0} \dfrac{e^x}{\frac{1}{\cos^2(2x)}\cdot 2} = \dfrac{1}{\frac{1}{1}\cdot 2} = \dfrac{1}{2}$



Pr6. $\lim\limits_{x\to \frac{\pi}{2}-} \overset{+\infty}{\text{tg}\, x} - \overset{+\infty}{\dfrac{1}{\cos x}} = \lim\limits_{x\to \frac{\pi}{2}-} \dfrac{\sin x}{\cos x} - \dfrac{1}{\cos x} = \lim\limits_{x\to \frac{\pi}{2}-} \dfrac{\overset{0}{\sin x - 1}}{\underset{0}{\cos x}} \overset{L'H}{=}$







$= \lim\limits_{x\to \frac{\pi}{2}-} \dfrac{\cos x}{-\sin x} = \dfrac{0}{-1} = 0$